The transient charm of decay

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Abstract. Undriven dissipative systems are investigated to find the best way to charaterize the chaotic decay towards a simple resting state. Traditional tools developed in regard to transient chaos are based on the presence of an infinity of unstable orbits, however, these are not present in undriven dissipative systems. We show that, instead of the recently proposed instantaneous, snapshot view of these systems, they must be observed at equal energy levels to find when the boundary between basins of attraction change from fractal to smooth.

Introduction

In undriven dissipative systems all motion decays since dissipation continually decreases the available mechanical energy. Chaotic motion can only show up transiently. Traditional transient chaos is, however, caused by the presence of an infinity of unstable orbits. In the lack of these, chaos in undriven dissipative systems is of another type: it is termed doubly transient chaos as the strength of transient chaos is diminishing in time, and ceases asymptotically [1]. To characterize the behavior of such systems, the snapshot view has been suggested [2, 3], but it does not lead to a clear characterization of e.g. the fractality of the boundary between the basins of attraction. We suggest that a view based on equal energy levels might be a better choice.

Results and discussion

A clear view of the dynamics of purely dissipative systems is provided by identifying KAM tori or chaotic regions of the dissipation-free case, and following their time evolution in the dissipative dynamics. The tori often smoothly deform first, but later they become disintegrated and dissolve in a kind of shrinking chaos. We show that using an equal energy level view instead of the usual snapshot view preserves the tori structure. Various dynamical measures can be utilized for the characterization of this process and they illustrate that the strength of chaos is first diminishing, and after a while disappears, the motion enters the phase of ultimate stopping. Meanwhile, with the decrease of the energy, the boundary between basins of attraction become gradually simpler, from fractal to smooth. However, to find a clear fractal property, basins of attraction must be observed at equal energy levels.

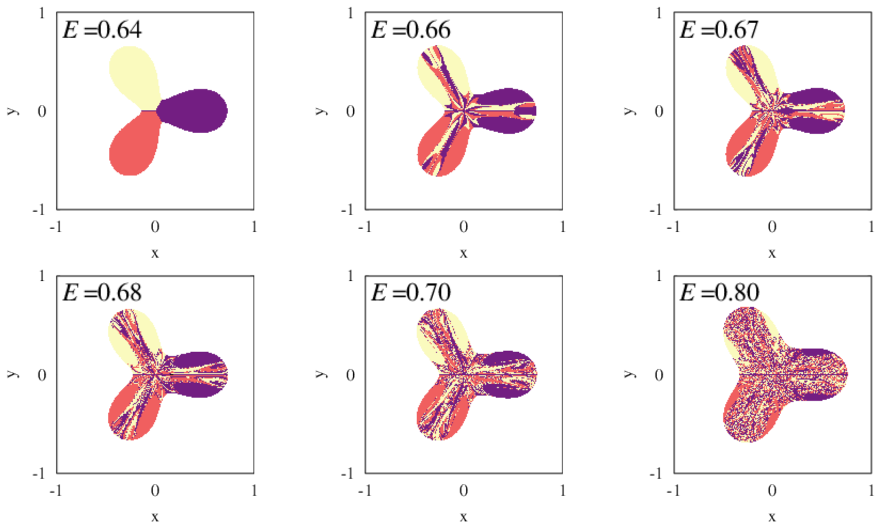

Figure 1: Basin boundary for the magnetic pendulum at equal energy levels.

References

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