Efficient path integral formulation for the response statistics of stochastic vibro-impact systems

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Abstract. We present an efficient formulation to obtain the time-dependent response probability density functions of vibro-impact oscillators for energy harvesting applications. We demonstrate the versatility of the proposed method through the analysis of time-invariant and time-dependent models of vibro-impact devices with energy harvesting and targeted energy transfer applications.

Introduction

Several systems in engineering are modelled by dynamical systems with impacts and random excitation [1]; e.g. a recently proposed energy harvesting system [2] uses dielectric elastomers that utilise vibro-impact motion to harvest energy from noisy ambient vibrations. One of the most important quantities describing the response of dynamical systems to random effects is the probability density function (PDF). Having access to a vibro-impact system's response PDF to noisy forcing allows the analysis of several important properties of the system. These properties include quantities such as n-th order moment evolution of states, the energy accumulated in the system, and other derived statistics such as the impact velocity PDF or mean energy absorbed by impacts. In general, obtaining these quantities is challenging even for a smooth nonlinear dynamical system subjected to noise. It is even more difficult if there are impacts in the system, such as in vibro-impact systems. However, there are processes and devices with impacts where the effects of noise play an essential role, and we need performant methods to compute their stochastic characteristics to investigate and understand their behaviour.

Results and discussion

In this talk, we present the generalisation of the novel step matrix-multiplication based formulation of the path integration (SMM-PI) method [3] for computing the response PDF of vibro-impact systems. The SMM-PI method is based on the law of total probability captured by the Chapman Kolmogorov (CK) equation, namely, we transform the CK equation to a matrix-vector multiplication utilising high-order numerical time stepping and interpolation methods. We use the SMM-PI method to compute quantities such as the response PDF, impact velocity distribution, energy accumulated and absorbed, and the mean time between impacts for different vibro-impact systems with a pair of impact barriers. We compare the results to Monte-Carlo simulations and show the superior ability of the SMM-PI formulation to compute accurate response PDFs.

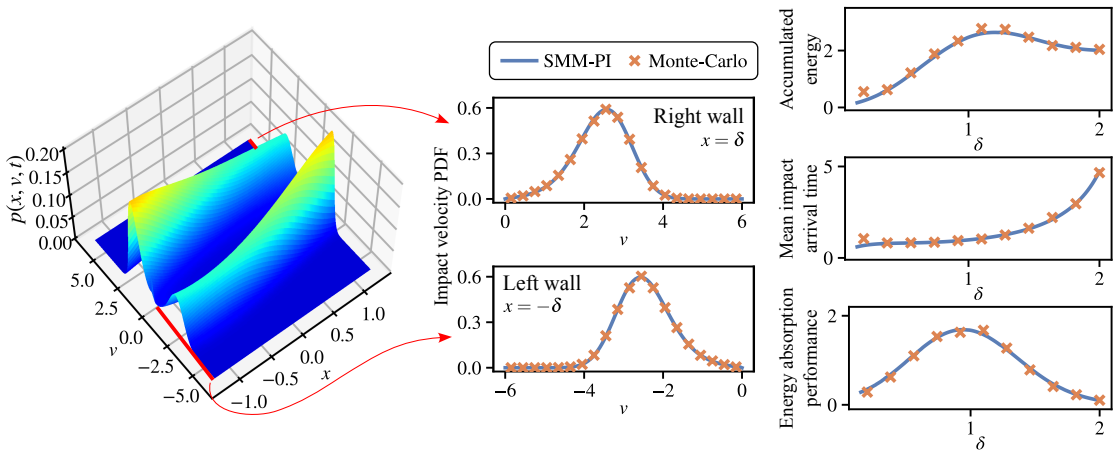

Figure 1: Probability density function, impact velocity distribution, and energy characteristics of a vibro-impact oscillator model with a pair of impact barriers.

References

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