Nonlinear free vibration of functionally graded shallow shells with variable thickness resting on elastic foundations

Kurpa Lidiya[*], Shmatko Tetyana[**], Awrejcewicz Jan[***], Timchenko Galina[*]

[*] *Department of Applied Mathematics, National Polytechnic University "KhPI", Kharkov, Ukraine*
[**] *Department of Higher Mathematics, National Polytechnic University "KhPI", Kharkov, Ukraine*
[***] *Department of Automation, Biomechanical and Mechatronics, Lodz University of Technology, Lodz, Poland*

Abstract. Geometrically nonlinear free vibration of the shallow shells of an arbitrary shape of the plan with variable thickness is studied. The shell is fabricated from functionally graded materials (FGM) and supported by elastic foundation. The refined shear deformation theory of the first order (FSDT) is used to obtain the theoretical formulation of the problem. Simple power law is applied to calculate the effective characteristics of the FGM in the thickness direction. Variation of the thickness of the shallow shells is carried out according to the given law: it can be linear, parabolic or another law. Elastic foundation is described by Pasternak's model. The proposed approach is based on application of the R-functions theory, variational Ritz's method, and procedure by Bubnov-Galerkin.

Introduction

From the analysis of the available literature, it can be concluded that there is a small number of publications in which the question of linear vibrations of the FGM shells with variable thickness, resting on an elastic foundation [1, 2], is studied. And there are practically no works on nonlinear vibrations of shallow shells of variable thickness, with complex shape in the plan. The present study focuses on these questions. The system of nonlinear equations of motion for FGM shallow shells is received using FSDT and Hamilton's principle. It is assumed that a plan form of the shell can be complex. Power law is used to calculate the effective material properties. Influence of the foundation is considered through relation $p_0 = K_w w - K_P \nabla^2 w$ [1].

Results and discussion

Proposed method is based on approach developed by authors earlier [3], which uses the R-functions theory [4] essentially. Validation of the suggested method and created software is made by comparison of the calculated results with known ones. New results for shallow shells with square and complex shape of the plan were obtained. For example, backbone curves for clamped (CL) and simply supported (SS) FGM (Al/Al$_2$O$_3$) square cylindrical shells $\left(a / R_x = 0.2, a / R_y = 0\right)$ with parabolic [2] $h(x, y) = h_0\left(1 - \alpha(x / a)^2\right)$ $(\alpha = 0.5)$ and constant thickness $(\alpha = 0)$ are presented in Fig 1, 2. Behaviour of the nonlinear frequencies is shown in Fig.3.

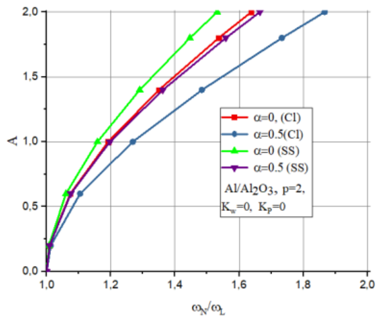

Figure 1: Backbone curves of cylindrical shell (K_w=0, K_p=0)

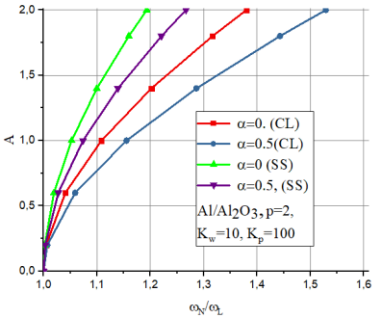

Figure 2: Backbone curves of cylindrical shell (K_w=10, K_p=100)

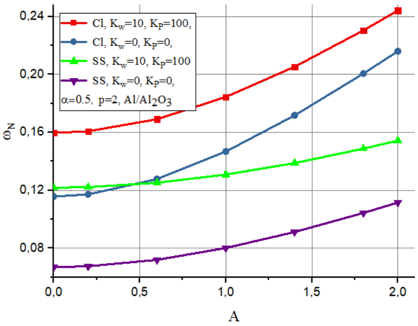

Figure 3: Nonlinear frequencies of cylindrical shell

As expected, the greatest increase in nonlinear frequencies is observed for clamped shells. Decreasing the parameter α increases the thickness of the shell and, consequently, increases the value of the frequencies.

References

[1] Shen H-S, Wang H. (2014) Nonlinear vibration of shear deformable FGM cylindrical panels resting on elastic foundations in thermal environments. *J. Composites:* Part B **60**: 167-177.

[2] Kumar V., Singh S.J., Saran V.H., Harsha S.P. (2022) Vibration response of shear deformable functionally graded material plate with parabolic variable thickness. IOP Conf. Series: *Mater. Sci. Eng.* **1248** 012069 doi:10.1088/1757-899X/1248/1/012069.

[3] Kurpa L., Shmatko T., Timchenko G. (2021) Nonlinear dynamic analysis of FGM sandwich shallow shells with variable thickness of layers. *Nonlinear Mechanics of Complex Structures*, Advanced Structured Materials, H. Altenbach et al. (eds.), **157**: 57-74.

[4] Rvachev V.L. (1982) The R-functions theory and its applications. Kiev: Nauk. Dumka (in Russian).