Parametric resonances due to torsional oscillations in a multi-degree of freedom driveline coupled by a series of universal joints

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Abstract. Here we study the parametric instabilities in a 3-degrees of freedom (DOF) driveline, consisting of a series of two universal joints (U-Joints) phased at 90^0. The governing non-linear equations of motion are linearized around a steady rotational motion of the drive shaft in relative coordinates representing rotational dynamics, and for the system with periodic coefficients stability is concluded using Floquet-Lyapunov theory. The existence of harmonic, sub-harmonic and combination-type resonances are identified on the scatter plot or Strutt diagram of the parameter plane. Linear stability analysis results are validated by direct time integration of the full non-linear model for stable and unstable parametric regions using the Runge-Kutta routine.

Introduction

The non-linear kinematical relationship between the input and output angular velocity of a U-Joint varies periodically resulting in a time-periodic behaviour of the dynamic system incorporating the U-Joint. Many studies have been conducted on such dynamic systems though the focus has mostly been on the development of equations of motion and their solutions rather than detailed dynamic behaviour. Bulut and Parlar [1] studied the dynamics of a 2-DOF shaft system interconnected with a U-Joint. The stability of the steady rotational motion of the system was determined using the monodromy matrix method and some parametric resonance regions below and above the fundamental torsional frequency of the system was highlighted. Asokanthan and Meehan [2] obtained conditions for parametric instability of the 2-DOF shaft system using the method of averaging, and determined the quasi-periodic route to chaos for the full nonlinear model utilizing maximal Lyapunov exponent. For the shaft system with a single U-Joint, they arrived at the necessary parametric and combination resonance conditions for chaotic behaviour. For a system with multiple U-Joint drivelines, a variety of challenges arise such as phasing a series of joints in the driveline to cancel the torsional oscillation effects due to the kinematics of U-Joint. Even with proper cancellation, the 2^{nd} Order torsional oscillations are always present in the driveline due to the inertial effects influenced by the inherent non-linearities. Also, in a real-world application, it is very difficult to keep the U-Joint misalignment angles equal for the series of joints in the driveline which invalidates the torsional cancellation effect completely even though joints are properly phased. Under certain operating conditions of the driveline, interesting non-linear resonance phenomena could be observed due to these uncancelled torsional and inertial oscillations present in the driveline. Therefore, it is important to investigate the dynamic stability of a shaft system consisting of multiple U-Joints with equal and unequal misalignment angles. It would be helpful from a design perspective to estimate critical resonance speeds and misalignment angles to prevent parametric or combination-type resonances in the driveline which are a product of the non-linearities of the dynamical system.

Results and Discussion

In this abstract, we have illustrated the parametric instabilities of a two U-Joints shaft system with equal joint misalignment angles $\beta_{1,2}$. Using Floquet theory for linear parametrically excited systems, stable and unstable zones are identified and presented in a strut diagram in $(\Omega, \beta_{1,2})$ plane where red dots represent the unstable region as shown in Figure 1. $(\Omega = 1)$ represents the fundamental torsional resonance and it can be inferred that for a small value of damping $(\zeta = 0.001)$. Direct simulations of the fully nonlinear system are also developed.

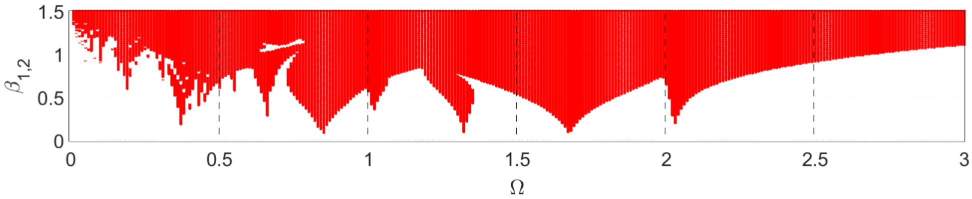

Figure 1: Stability chart - Equal misalignment angle $\beta_1 = \beta_2$

References

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[2] S.F. Asokanthan, P.A. Meehan, Non-linear vibration of a torsional system driven by a Hooke's joint, Journal of Sound and Vibration, Volume 233, Issue 2, 2000, Pages 297-310