Locomotion dynamics of an underactuated wheeled three-link robot

Leonid Rizyaev, Yizhar Or

Faculty of Mechanical Engineering, Technion – Israel Institute of Technology, Haifa, Israel

Abstract – The wheeled three-link snake robot is a well-known example of an underactuated system modelled using nonholonomic constraints, preventing lateral slippage of the wheels. A kinematically controlled version assumes that both joint angles are directly prescribed as phase-shifted periodic input. In another version of the robot, only one joint is periodically actuated while the second joint is passively governed by a visco-elastic torsion spring. In our work, we constructed the two versions of the wheeled robot and conducted motion experiments under different actuation inputs. Analysis of the motion tracking measurements revealed significant amount of lateral slippage, in contrast to the standard nonholonomic models. Therefore, we proposed modified dynamic models which include wheels' lateral slippage and viscous friction forces, as well as rolling resistance. After parameter fitting, these dynamic models reach good agreement with the motion measurements, including effects of input's frequency on the mean speed and net displacement per period. This illustrates the importance of incorporating slippage and friction into the system's model.

Introduction

Underactuated multi-link robot locomotion has been addressed in many literature studies. One of the most classic examples is the wheeled three-link snake robot, composed of three rigid links supported by wheels and connected by rotary joints. Ideally, the wheels are assumed to resist lateral slippage, which induces non-holonomic constraints on the robot's motion [1]. These types of systems are called underactuated since their locomotion is generated by changing the shape, rather than directly controlling the body variables. A later work [2] studied the dynamics of the kinematically controlled snake robot near its singular symmetric states where $\phi_1 = \phi_2$. Regarding the semi-passive actuation, the work [3] focused on analysing the vehicle's dynamics with a single actuated joint, while the other joint is passive, governed by visco-elastic torsion spring, applying torque as $\tau_1 = -k\phi_1 - c\dot{\phi}_1$. Our work extends the previous studies both theoretically and experimentally. We analyzed the motion of the three-wheel snake (Fig. 1a) in both shape-actuated and semi-passive configurations. We conducted motion tracking measurements of our robotic prototype in both configurations, measuring the influence of input frequency on the motion, both in asymmetric input gaits, and symmetric ones that cross singular configurations.



Figure 1: (a) The three-link robot model. (b) Our three-link robot with kinematic actuation at both joints. (c) Comparison with simulation results of different models with and without roll/slip dissipation for the of the kinematically-actuated robot.

Results and core findings

Motion measurements for the shape-actuated configuration show significant influence of actuation frequency on the mean speed and displacement (Fig. 1c), which cannot be explained by the kinematic equation of nonholonomic constraints based on no-slip conditions $v_i^{\perp}=0$. This effect is caused by non-negligible lateral slippage of the wheels, due to inertial effects, as well as nearing singularities. To account for slippage, and to avoid non-smoothness and numerical sensitivity incurred by Coulomb-type friction [2], we incorporated linear viscous damping friction forces, as $f_i = -c_s v_i^{\perp}$ for each axle. In addition, rolling resistance of the wheels is also represented as linear damping $f_j^{\parallel} = -c_r v_i^{\parallel}$ for each wheel. After calibration of the roll/slip damping coefficients, we obtained good quantitative agreement with the experimental results (Fig. 1c). For the semi-passive configuration, the model predicts dependence on actuation frequency, even with no-slip constraints, and the effect of slip dissipation is increased for symmetric actuation which passes near singular states.

References

- [1] Shammas, E. A., Choset, H., & Rizzi, A. A. (2007). Geometric motion planning analysis for two classes of underactuated mechanical systems. *The International Journal of Robotics Research*, 26(10), 1043-1073.
- [2] T. Yona and Y. Or, "The wheeled three-link snake model: singularities in nonholonomic constraints and stick-slip hybrid dynamics induced by Coulomb friction," Nonlinear Dynamics., vol. 95, no. 3, pp. 2307–2324, 2019.
- [3] T. Dear, S. D. Kelly, M. Travers, and H. Choset, "Locomotive analysis of a single-input three-link snake robot," 2016 IEEE 55th Conf. Decis. Control. CDC 2016, no. Cdc, pp. 7542–7547, 2016.