Expanding measure has nonuniform specification property on random dynamical system

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Abstract. In the present paper, we study the distribution of the return points in the fibers for a random non uniformly expanding dynamical system, preserving an ergodic probability. We also show the abundance of nonlacunarity of hyperbolic times that are obtained along the orbits through the fibers. We conclude that any ergodic measure with positive Lyapunov exponents satisfies the nonuniform specification property among fibers. As consequences, we prove that any expanding measure is the limit of probability measures whose measures of disintegration on the fibers are supported on a finite number of return points and we prove that the average of the measures on the fibers corresponding to a disintegration, along the orbit $(\theta^n(w))_{n\geq 0}$ in the base dynamics is the limit of Dirac measures supported on return orbits on the fibers.

Introduction

There exist various generalizations of the specification property and among them are almost specification, relative specification, periodic weak specification, weak specification, relative weak specification. The connection among them can be found in [1]. These kinds of specification give tools of great utility in exploring the topological structure and statistical behavior of some systems.

On the other hand, when we investigate the strong specification property of trajectories by periodic orbits, we realize that it is very strange to happen in the RDS (random dynamical system) context. It is worth asking, given a random dynamics $F : X \times M \to X \times M$ defined as $F(w, x) = (\theta(w), f_w(x))$, where $\theta : X \to X$ is an invertible map and the $f_w : M \to M$ are local homeomorphism for all $w \in X$ when a point in the fiber dynamics repeats in its orbit i.e. given $p \in M$ it is possible that $p \in \{f_w^n(p) := f_{\theta^{n-1}(w)} \circ \cdots \circ f_{\theta(w)} \circ f_w(p) \ : \ for\, n \geq 1\}$.

To obtain this result, we consider a non-uniformly expanding random dynamics which is a system that has already been studied by many authors. For example, the work by J. Alves and V. Araújo [2], gives necessary and sufficient conditions for the stochastic stability of nonuniformly expanding maps either with or without critical sets. Arbieto, Matheus and Olivera [3] show the existence of equilibrium states for some random non-uniformly expanding maps.

A peculiarity of some of these systems is the existence of hyperbolic times, which is crucial to prove that our system has the property of weak specification. Furthermore, with the existence of hyperbolic times along an orbit through the fibers, we show that this sequence of hyperbolic times is non-lacunary. Hyperbolic times allow us to deduce that the system is expansive, a property that is reflected by measuring the topological entropy of the system see [4, 5]. To obtain the points of return on the fibers, we start with the existence of an expanding measure for the non-uniformly expanding random system. Now, on the set of total measure with respect to the expansive measure, we show that each orbit along the fibers has a sequence of hyperbolic times and also satisfies the non-lacunary property. With these two new features, we show that this type of system has the non-uniform weak specification property. Then, based on this specification property, we manage to build the return points on the fibers.

As a consequence of this specification result, we have that

- The expanding measure is the limit of invariant measures whose measures of their disintegrations on the fibers are supported on a finite number of return points.
- The average of the measures on the fibers corresponding to a disintegration, along an orbit in the base dynamics, is the limit of Dirac measures supported on return orbits on the fibers.

References

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