# The effects of road curvature on the stability of path-following of automated vehicles

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**Abstract**. The stability analysis of a path-following controller for automated vehicles is presented, with the consideration of path curvature and feedback delay. The analysis is based on a kinematic vehicle model expressed in a path reference frame. The steering controller includes feedforward and feedback terms, and the time delay in the control loop is also considered. The effects of parameters such as the vehicle speed and the path curvature are analyzed using compact analytical expressions and stability charts. The results help the designer select the optimal control gains and the limitations of tuning the controller with the assumption of a straight-line reference path are also shown.

## Introduction

Designing stable and safe path following controllers with sufficiently high performance is a crucial step towards vehicle automation and it is also a cornerstone of many advanced driver assistance systems, such as lane-keeping and lane changing control. There are many different approaches to design the corresponding controllers, from simple geometrical considerations to machine learning-based methods [1].

In this study, the analysis of a steering controller for path following is presented, with the consideration of time delay in the control loop and the curvature of the reference path. The calculations are based on a kinematic single-track vehicle model, which allows us to present the results in the form of compact analytical expressions. The lateral constraint forces at the wheels are also calculated to ensure that loss of traction does not occur. Using a coordinate transformation, the vehicle model is transformed from the global coordinate system to a path reference frame (see Fig. 1(a)), so that the vehicle motion is described with respect to the reference path [2].

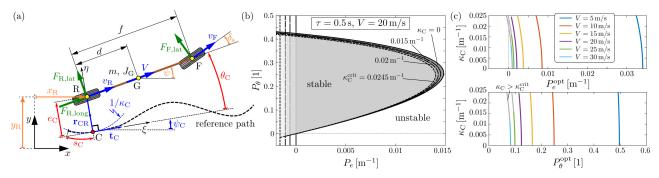


Figure 1: (a) Vehicle model in path reference frame. (b) Stable domains of control gains  $P_e$  and  $P_{\theta}$  depending on the path curvature  $\kappa_{\rm C}$ . (c) Optimal control gains in terms of fastest decay for different values of path curvature and vehicle speed.

#### **Results and discussion**

In order to achieve stable path following, the steering angle is generated using a combination of feedforward and feedback control. The feedforward term is used to determine the ideal steering angle corresponding to the path curvature based on the kinematics of the vehicle, while the feedback controller ensures the stability of tracking the reference path. The feedback action is based on the lateral deviation  $e_{\rm C}$  and the angle error  $\theta_{\rm C}$  of the vehicle with respect to the reference path, and the feedback delay  $\tau$  in the control loop is also considered. The stability analysis of the controlled vehicle is performed by linearizing the system along the equilibrium of stable path following with zero tracking error. Analytical expressions are then derived for the stability boundaries of the linearized system. This allows us to gain a deeper understanding of how certain parameters affect stability and how the control gains should be adjusted depending on the vehicle speed and the curvature of the reference path (see Fig. 1(b)). The optimal control gains that lead to the fastest decay of the linearized system are also determined and analyzed (Fig. 1(c)). In particular, we show the limitations of tuning the

# system are also determined and analyzed (Fig. 1(c)). In particular, we show the limitations of tuning the controller with the assumption of a straight-line reference path and the safe ranges of vehicle speed and path curvature for a given controller are also determined.

## References

- Paden, B., Čáp, M., Yong, S. Z., Yershov, D., Frazzoli, E. (2016) A survey of motion planning and control techniques for selfdriving urban vehicles. *IEEE Transactions on Intelligent Vehicles*, 1(1):33-55.
- [2] Qin, W. B., Zhang, Y., Takács, D., Stépán, G., Orosz, G. (2022) Nonholonomic dynamics and control of road vehicles: moving toward automation. *Nonlinear Dynamics*, 1-46.