# Resource Sensitive Game

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**Abstract**. The idea of resource sensitive game is proposed to address the evolutionary dynamics in multi-agent dynamical systems subjected to fluctuating resource. The co-evolution between a specifically define resource function and the players' payoff provides the key in our formulism. Numerical study based on the classical hawk-dove game is provided with momentum and contrarian hawkish player behaviours. Rich evolutionary dynamics with periodic and chaos is shown.

## Introduction

Game theory provides the conceptual framework for studying multi-agent dynamical system with self-interest players [1]. The classical game models assume a constant player's payoff as well as an unlimited resource from which the payoffs are drawn. This is justifiable when the environment acts as a passive player who does not interact with the game. In an environment causing payoff variation, anomalous evolutionary stable states and sub-optimality in game dynamics were both reported [2]. The carbon metabolism in yeast cells provide a different scenario of an environment subjected to fluctuating resources, where 'cheater' strain yeast cells, coexisting with their cooperative counterparts in the population, can take a free ride with the glucose they need but never produce [3,4]. Gore et al. show that the underlying dynamics must necessarily exhibit hybrid characteristics of the prisoner-dilemma and hawk-dove games [3]. Indeed, from a predator-prey system to the so-called *tragedy of the commons* in general, the possibility of a more complex evolutionary dynamics has long been suggested when the players' objectives intertwined with the environment [4,5]. However, both the modelling of this player-resource interaction and its consequence in the evolutionary dynamics have not received sufficient attention in the past.

In this work, the idea of resource sensitive (RS) game is proposed. A time co-evolution of the players' payoff and the system resource is introduced to reveal the evolutionary complexity that is yet to be addressed by the existing game theory. We demonstrated the idea using the classical hawk-dove game with the payoff matrix $B = \begin{bmatrix} (b_n^h - c)/2 & b_n^h \\ 0 & b^d/2 \end{bmatrix}$. Both momentum and contrarian hawkish behaviours are considered in the co-evolution with the system resource. The momentum player 'doubles down' in a rising resource environment whereas the contrarian acts in the opposite. The dynamical system of equations can be given by $x_{n+1} = x_n(b_n^h - (b_n^h + c)x_n/2)/A(n)$, $b_{n+1}^h = b_0^h + \epsilon[\tan^{-1}(\gamma\omega_n) + \pi/2]/\pi$,$\sigma_{n+1} = \sigma_n + d(b_n^h, c, b^d, \mu)x_n^2 + h(b_n^h, c, b^d, \mu)x_n + k(b_n^h, c, b^d, \mu)$, where $x$ is the percentage of the hawkish population, $A(n)$, the averaged payoff, $\mu$ and $\epsilon$, real constants in [0,1], $\omega_n = (\sigma_n - \sigma_0)/\max(\sigma_1 - \sigma_0, \ldots, \sigma_{n-1} - \sigma_0)$ and $d, h, k$ are coefficients for the quadratic law of the resource $\sigma_n$. In addition to the classical evolutionary stable states, the RS hawk-dove game exhibits periodic and chaotic evolutions; see Fig. 1. The chaotic solution is particularly interesting in that a higher concentration of resource (wealth) in a smaller hawkish population is observed when the resource level rises, a sort of a rich-get-richer scenario.

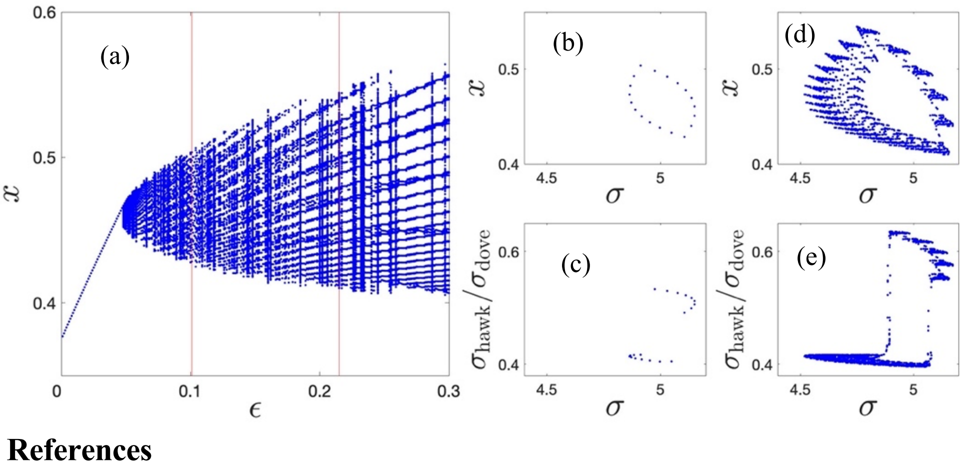


Fig. 1: Numerical simulation of the RS hawk-dove game with momentum hawk: (a) the hawk population $x$ vs. $\epsilon$. The $x$ and hawkish-dovish wealth ratio, $\sigma_{\text{hawk}}/\sigma_{\text{dove}}$, vs. the resource level $\sigma$, respectively, in (b), (c), showing the periodic solution at $\epsilon = 0.101$, and (d), (e), showing the chaotic solution at $\epsilon = 0.215$. Simulation parameters: $\gamma = 900$, $\sigma_0 = 5$, $c = 1$, $b_0^h = 0.5$, $b^d = 0.7$, $\mu = 0.25$.

## References

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