Amplitude-Voltage Response of Superharmonic Resonance of Fourth Order of Electrostatically Actuated MEMS Cantilever Resonators

Dumitru I. Caruntu* and Christopher I. Reyes*

Department of Mechanical and Aerospace Engineering, University of Texas Rio Grande Valley, Edinburg, TX, USA

Abstract. This paper deals with amplitude-voltage response of electrostatically actuated MEMS cantilever resonator sensors undergoing a superharmonic resonance of fourth order. The system consists of a MEMS cantilever parallel to a ground plate and under an alternating current (AC) producing a hard excitation. The AC frequency is near one eighth of the first natural frequency of the cantilever. This leads to superharmonic resonance of fourth order (or order four). The electrostatic force includes fringe effect and it is modeled using Palmer's formula. The methods used in this work are 1) the method of multiple scales (MMS), 2) homotopy analysis method (HAM), and 3) numerical integration of reduced order model (ROM) using 2 modes (terms) of vibration. The amplitude-voltage response shows a softening effect and two saddle-node bifurcation points. All three methods are in agreement for moderate dimensionless voltage parameter (less than 0.8). The effects of damping, fringe, and detuning frequency on the voltage response are reported. As the damping increases the peak amplitude decreases. As fringe and/or detuning frequency increase the saddle-node bifurcation points shift to lower voltages.

Introduction

This work investigates a MEMS cantilever resonator undergoing superharmonic resonance of order four. This superharmonic resonance case requires hard excitations where the dimensionless voltage applied is greater than 0.5, otherwise the response is too small. MEMS cantilevers have gained in interest over the last couple of decades due to their potential applications in switches, sensors, filters, resonators, energy harvesters, micro probes, and micro pumps. These systems are advantageous in that they have a low fabrication cost, low energy requirement, are light weight, and can be placed in virtually any system due to their micro size. Superharmonic resonances of second order for MEMS devices have been reported in the literature [1].

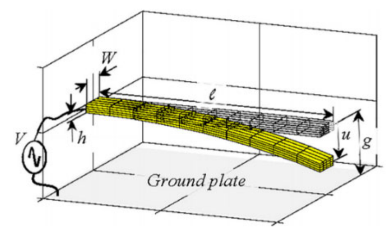

Figure 1: MEMS cantilever

In this work the MEMS cantilever is modeled as an Euler-Bernoulli beam, Fig. 1. The frequency of the AC voltage applied between the MEMS cantilever and the ground plate is near one eighth of the first natural frequency of the cantilever beam. The electrostatic force actuating the MEMS cantilever includes the fringe effect which is an additional electrostatic force outside of the control volume between the parallel plates, Palmer formula [2].

Results and discussion

Figure 2 shows the amplitude-voltage response using MMS, HAM, and 2T ROM time responses. The MMS, HAM and 2T ROM time responses are in agreement. The two saddle-node bifurcation points *A* and *B* predicted by MMS and HAM, are not contradicted by the time responses. However, time responses predict larger amplitude and lower voltage for the bifurcation point *B*. Also, time responses predict a slightly greater softening effect. As the voltage is sweep up, the system experiences stability and a steady increase in amplitude until the bifurcation point *A* is reached. At this point the system loses stability and jumps to a slightly higher amplitude on the second stable branch. As the voltage is continued to increase the system decreases in amplitude reaching the non-resonant region where in the amplitudes begin to slowly increase. As the voltage is swept down the system experiences stability and an increase in amplitude until reaching the bifurcation point B where the system loses stability and jumps to a lower amplitude. As the voltage decreases to zero, the amplitude decreases to zero.

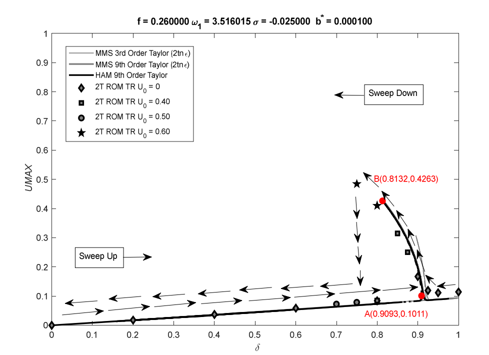

Figure 2: Amplitude (*Umax*) – voltage (*delta*) response

References

[1] Caruntu D., Botello M., Reyes C., Beatriz J. (2019) Voltage-Amplitude Response of Superharmonic Resonance of Second Order of Electrostatically Actuated MEMS Cantilever Resonators. *J. Comput. Nonlin. Dyn.* **14**: 031005 (8 pages).

[2] Batra R.C., Porfiri M., Spinello D. (2006) Electromechanical Model of Electrically Actuated Narrow Microbeams. *J. Microelectromech. Syst.* **15**(5): 1175–1189.