Stability control of two-wheeled trailers

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Abstract. The nonlinear dynamics of towed two-wheeled trailers is investigated. A spatial 4-DoF mechanical model is used, where all the yaw, pitch and roll motions are considered. Geometrical nonlinearities and the non-smooth characteristics of the tire forces are taken into account. A possible control algorithm is analyzed, which actuates by means of braking to one of the wheels of the trailer. Numerical bifurcation analysis is performed in order to investigate the large amplitude vibrations and unsafe (bistable) zones, where the stable rectilinear motion and the stable limit cycle coexist. It is shown, that with appropriately chosen control gains, the size of the bistable region can be decreased.

Introduction

Vehicle handling and stability are critical factors when investigating the so-called snaking motion of trailers. Most of the previous studies are limited to linear stability analysis and/or are based on in-plane models. Here, we investigate the nonlinear dynamics of two-wheeled trailers with a spatial mechanical model. The applied 4-DoF mechanical model is shown in Fig. 1(a), for which the governing equations are summarized in [1].

In order to reduce the unwanted vibrations of the system, stability control is applied, namely, braking forces are driven to the wheels. In [2], the effect of the braking forces is emulated via a control moment. Here, a more reliable model is analyzed, where stability is achieved via braking forces that are proportional to the yaw rate $\dot{\psi}$, see panel (a) of Fig. 1. A deadzone of the controller is also considered, where no braking force is actuated. The non-smooth characteristics of the right and the left braking forces can be formulated as

$$F_{\mathrm{R}}^{\mathrm{brake}} = \begin{cases} D(\dot{\psi} - \dot{\psi}_0)\,, & \text{if} \quad \dot{\psi} > \dot{\psi}_0 \\ 0\,, & \text{if} \quad \dot{\psi} < \dot{\psi}_0 \end{cases} \qquad F_{\mathrm{L}}^{\mathrm{brake}} = \begin{cases} -D(\dot{\psi} + \dot{\psi}_0)\,, & \text{if} \quad \dot{\psi} < \dot{\psi}_0 \\ 0\,, & \text{if} \quad \dot{\psi} > \dot{\psi}_0 \end{cases}\,, \tag{1}$$

where D is the control gain for the yaw rate and $\dot{\psi}_0$ is the deadzone of the controller.

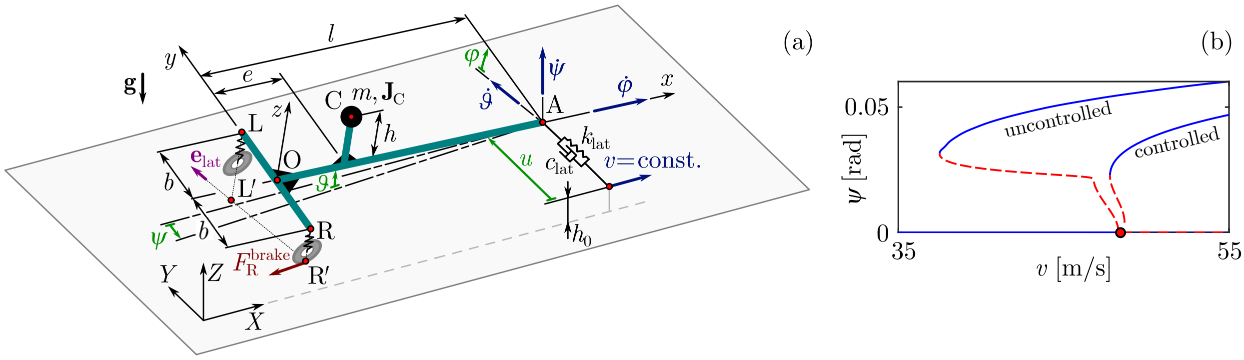

Figure 1: The spatial, 4-DoF mechanical model of towed two-wheeled trailers with the braking force for the right wheel (a), bifurcation diagrams of the uncontrolled and the controlled system for the yaw angle ψ with respect to the towing speed v (b).

Results and discussion

Nonlinear bifurcation analysis of the system is performed with the help of *DDE Biftool* [3], where the non-smooth characteristics of the tire forces and the braking forces are handled by a smoothed version of the Heaviside-function. The bifurcation diagrams of the uncontrolled and the controlled system are depicted in Fig. 1(b) for parameter values described in [1] and for the vertical payload position $h = 0.27\,\mathrm{m}$. Since the controller has a deadzone, the critical speed of the linear stability boundary (where Hopf bifurcation takes place) does not change due to the stability control. However, the unsafe zone (i.e. the bistable region) is reduced compared to the uncontrolled case.

As a future work, the feedback delay of the controller could be taken into account to tune the control gain with respect to the optimal performance of the controller.

References

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