Lie symmetry classification and exact solutions for a class of diffusive SIS epidemic model

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Abstract. We investigate a class of diffusive SIS epidemic model in Lie symmetry classification perspective. A Lie group classification is provided for different forms of infection mechanism $\Phi(S, I)$. We establish the Lie symmetry determining equations by utilizing well-known computer packages from literature. The Lie symmetries are provided for different forms of $\Phi(S, I)$. For the standard incidence $\Phi(S, I)$, the optimal system of one-dimensional subalgebras deduced from the Lie point symmetries of the model is then used to construct reductions and group-invariant solutions.

Introduction

Several mathematical models for studying the transitional dynamics of infectious disease have been developed. Nonlinear differential equations (DEs) are useful tools for determining the persistence or extinction of disease. We study the following diffusion-based SIS epidemic model:

$$\begin{aligned} S_t &= \theta S_{xx} - \beta\Phi(S, I) + \gamma I, \\ I_t &= \theta I_{xx} + \beta\Phi(S, I) - \gamma I, \end{aligned} \tag{1}$$

where $S(t, x)$ denote the susceptible, $I(t, x)$ denote infected individuals, $\Phi(S, I)$ represents the infection mechanism, $\beta > 0$ denote the infection rate, $\gamma > 0$ is the recovery rate, $N = S + I$ is the total population and θ is diffusion rate. We study the diffusive SIS epidemic model for exact solutions from the Lie point symmetry perspective.

Results and Discussion

A Lie group classification is provided for different forms of infection mechanism $\Phi(S, I)$. We establish the Lie symmetry determining equations by utilizing the computer packages [1, 2, 3]. The results of Lie symmetry classification are summarized in the Table 1.

For arbitrary incidence $\Phi(S, I)$	$X_1 = \partial_t, X_2 = \partial_x$
For mass action incidence $\Phi(S, I) = SI$	$X_1 = \partial_t, X_2 = \partial_x, X_3 = t\partial_t + \frac{1}{2}x\partial_x + (\frac{\gamma}{\beta} - S)\partial_S - I\partial_I$
For standard incidence $\Phi(S, I) = \frac{SI}{S+I}$	$X_1 = \partial_t, X_2 = \partial_x, X_3 = S\partial_S + I\partial_I, X_4 = 2\theta t\partial_x - xS\partial_S - xI\partial_I$
For saturated incidence $\Phi = \frac{SI}{1+mI}$	$X_1 = \partial_t, X_2 = \partial_x$

Table 1: Lie symmetries for different forms of infection mechanism $\Phi(S, I)$

For the standard incidence $\Phi(S, I) = \frac{SI}{S+I}$, the optimal system of one-dimensional subalgebras spanned is by

$$X_2,\ X_3,\ X_4 + \lambda X_1,\ X_1 + \mu X_3, \tag{2}$$

where λ and μ are constants. We then construct reductions and group-invariant solutions of diffusive SIS model (1) with the aid of the optimal system of one-dimensional subalgebras provided in equation (2).

References

[1] Rocha Filho, T. M., & Figueiredo, A. (2011) [SADE] a Maple package for the symmetry analysis of differential equations. Comput. Phys. Commun **182**: 467-476.

[2] Champagne, B., Hereman, W., & Winternitz, P. (1991) The computer calculation of Lie point symmetries of large systems of differential equations. Comput. Phys. Commun **66**: 319-340.

[3] Hereman, W. (1993) SYMMGRP. MAX and other symbolic programs for lie symmetry analysis of partial differential equation. Lect. appl. math **29**: 241-257.