Harmonic and superharmonic components in periodic waves propagating through mechanical metamaterials with inertia amplification

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Abstract. In the nonlinear dynamics of periodic microstructured systems, the amplitude-dependent dispersion properties of mechanical metamaterials are attracting increasing interest. The paper investigates the free harmonic and superharmonic response of a non-dissipative one-dimensional waveguide with pantographic mass amplification. The effects of the quadratic and cubic inertia nonlinearities on the dispersion curve are analyzed. A perturbation strategy based on the Fourier series decomposition is adopted to determine the superharmonic amplitudes of the propagating periodic waves.

Introduction

The dispersion properties of microstructured periodic waveguides are a scientific topic of major interest in nonlinear dynamics. Particularly, the long-dating tradition of research studies concerning the propagation of nonlinear elastic waves in oscillator chains and other periodic structures [1, 2] is being catalyzed by the recent extraordinary development of microstructured materials and mechanical metamaterials [3]. Specifically, a pressing technological demand is boosting the innovation in the emerging field of architected passive media. In order to broaden the existing range of achievable functionalities, the periodic cell of mechanical metamaterials is often enriched with auxiliary flexible microstructures inducing relevant linear and nonlinear dynamic phenomena, like local resonances, controlled instabilities, gyroscopic couplings, inertia amplifications [4, 5].

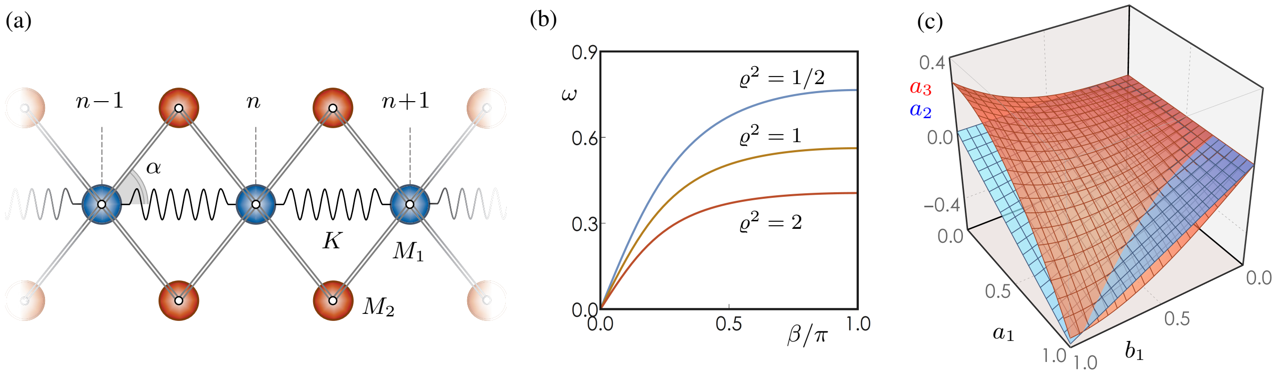

Figure 1: Mechanical metamaterial: (a) periodic atomic chain, (b) linear dispersion curves, (c) superharmonic amplitudes.

Results and discussion

A minimal physical system featured by a periodic pantographic mechanism of inertial amplification can be realized by an infinite one-dimensional non-dissipative chain of interconnected massive atoms (Figure 1a). In the periodic pantographic mechanism, two point masses (*primary blue atoms*) are elastically coupled to each other and rigidly connected to a pair of auxiliary point masses (*secondary red atoms*). The free undamped dynamics of the atomic chain is governed by a second-order differential (in time) difference (in space) equation with quadratic and cubic inertia nonlinearities. Attention is paid to the free propagation of harmonically periodic waves. To this purpose, the solution of the governing equation is expressed in a time-space Fourier series involving harmonic (ω) and superharmonic ($2\omega, 3\omega$) time components. Considering first the range of small oscillations, the amplitudes of the Fourier components are ordered in power series of a small perturbation parameter. Solving the resulting perturbation equations allows the parametric assessment of (i) the linear dispersion curve at the lowest order, exhibiting a marked dispersive behavior (Figure 1b), (ii) the nonlinear amplitude-dependent dispersion curve and (iii) the superharmonic amplitudes at the higher orders (Figure 1c).

References

[1] Vakakis, A.F., King, M.E. (1995). Nonlinear wave transmission in a monocoupled elastic periodic system. *J. Acoust. Soc. Am.* **98**(3): 1534-1546.

[2] Romeo F., Rega G. (2006). Wave propagation properties in oscillatory chains with cubic nonlinearities via nonlinear map approach. *Chaos Solitons Fractals* **27**(3): 606-617.

[3] Hussein, M.I., Leamy, M.J., Ruzzene, M. (2014). Dynamics of phononic materials and structures: Historical origins, recent progress, and future outlook. *Appl. Mech. Rev.* **66**(4): 040802.

[4] Settimi, V., Lepidi, M., Bacigalupo, A. (2021). Nonlinear dispersion properties of one-dimensional mechanical metamaterials with inertia amplification. *Int. J. Mech. Sci.* **201**: 106461.

[5] Settimi, V., Lepidi, M., Bacigalupo, A. (2023). Analytical spectral design of mechanical metamaterials with inertia amplification. *Eng. Struct.* **274**: 115054.