Effect of Boundary Conditions on the Stability of a Viscoelastic Von Mises Truss

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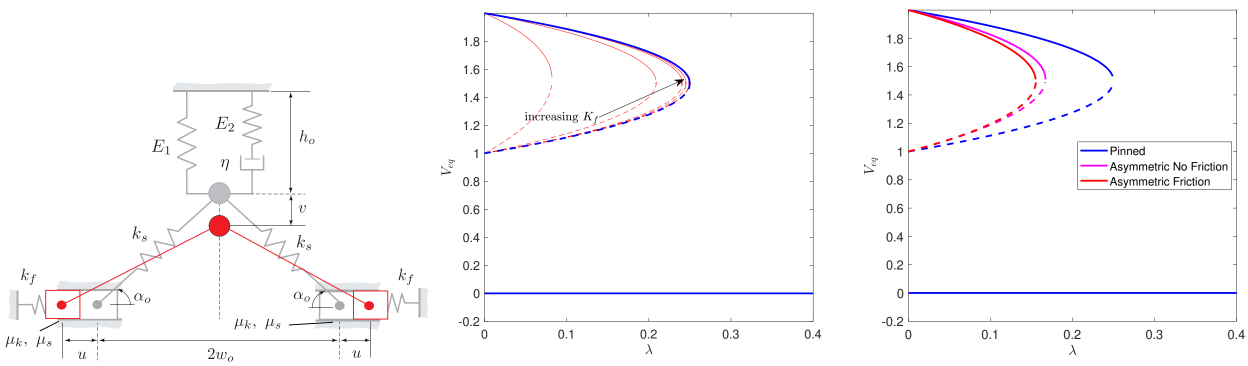
Abstract. This research aims to analyze the stability of bi-stable dome-shaped structures represented as a lumped parameter viscoelastic Von Mises Truss under different boundary conditions. Traditionally, the stability of these lumped parameter systems has been studied under pin-pinned boundary conditions. However, a closer investigation of the deformation of the dome reveals that the contact between the dome and the surface is not fixed. Instead, it slides outwards as the dome is indented. This phenomenon raises the question of whether the stability will alter if the boundary conditions are altered. With this motivation, we explore two alternate boundary conditions: 1) symmetric and 2) asymmetric. We demonstrate how relaxing or constraining the contact surface leads to change in the bifurcation point. This work allows us to gain some fundamental insights in modeling and controlling the spatiotemporal behavior of biological and engineering structures.

Introduction

The study of viscoelastic domes has mainly been limited to either experiments or finite element simulations. Recently, Gomez et al. [1] studied the stability and dynamics of a viscoelastic dome represented as a lumped parameter Von Mises truss. They analyzed the dynamics in the pseudo-bistable region using the method of multiple scales and derived an analytical expression for the snapping time. Liu et al. [2] used a similar lumped parameter system to study the effect of dimple-shaped imperfections on the stability of viscoelastic domes. However, most treatments of these systems using lumped parameter models consider pin-pinned boundary conditions and do not take into account the sliding of the dome as it is indented. If the dome were to be connected to another structure, then the manner in which the joint between the dome and the structure is implemented would dictate the boundary conditions of the dome. This motivates us to explore alternate boundary conditions. Here we study two alternate boundary conditions: 1) where both the pin joints in the conventional system are replaced by sliders (Figure 1(a)), and 2) an asymmetric boundary condition, where only one of the pin joints is replaced with a slider. We also study how friction at the sliders influences the stability of the system.



(a) (b) (c)

Figure 1: (a) Schematic of Von Mises Truss in undeformed or natural configuration (gray) and deformed configuration (red), (b) Equilibrium points (V_{eq}) of system in (a), plotted against the bifurcation parameter (λ), i.e. the relative stiffness between the viscoelastic element and the truss's central springs, for various horizontal spring stiffness (K_f), and (c) Bifurcation diagram for the system with asymmetric boundary condition, with and without friction, compared against the conventional system with pin-pinned boundary condition. Solid lines indicate stable equilibrium points. Dashed lines indicate unstable equilibrium points.

Results and Discussion

The parameter value at bifurcation (λ_b) varies non-linearly with the horizontal spring stiffness (K_f). With increase in horizontal spring stiffness, the equilibrium points converge to those of the conventional system as shown in Figure 1(b). The influence of the asymmetric boundary condition on the equilibrium points of the truss is shown in Figure 1(c). We find that when the boundary conditions are relaxed, the bifurcation occurs at a lower parameter value. Friction at the sliders also causes the system to bifurcate at a lower parameter value.

References

[1] Gomez M., Moulton D.E., Vella D. (2019) Dynamics of viscoelastic snap-through. *J. Mechanics and Physics of Solids* **124**:781-813.

[2] Liu T., Chen Y., Liu L., Liu Y., Leng J., Jin L. (2021) Effect of imperfections on pseudo-bistability of viscoelastic domes. *Extreme Mechanics Letters* **49**:101477.