Existence and irreproducibility of ultra-chaos

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Abstract. This work illustrates that the ultra-chaos exists in some chaotic dynamical system. For normal-chaos, although small disturbances lead to large deviations of trajectories, they do not affect the statistics. In contrast to a normal-chaos, the statistical characteristics of an ultra-chaos like probability density functions (PDF), are sensitive to small disturbances. It is shown that statistical irreproducibility is a fundamental characteristic of an ultra-chaos. Therefore, since random environmental noises always exist and are uncontrollable, it is indeed difficult in practice to replicate results of an ultra-chaos, even from statistical perspective. The ultra-chaos might open a new door and possibility to study chaos theory.

Introduction

The chaos theory is widely considered as one of the greatest scientific revolution in physics in 20th century. Poincaré [1] first discovered the sensitivity dependence on initial condition of a chaotic system. This results was rediscovered by Lorenz [2] with a well-known name "butterfly-effect". furthermore, Lorenz [3] discovered that the chaotic systems is sensitive to numerical algorithms. For the purpose of gaining reliable numerical simulations of chaotic results, Liao [4] suggested a numerical strategy, namely the "Clean Numerical Simulation" (CNS). Within the frame of the CNS [4], the spatial and temporal truncation errors and round-off errors can be decreased to a required small level so that the numerical simulation can be reliable within the spatial domain for a long enough time. The damped driven sine-Gordon equation is a well-known spatio-temporal chaos. Therefore, by means of the CNS, we can accurately investigate the influence of the small disturbances on the statistics for the damped driven sine-Gordon equation.

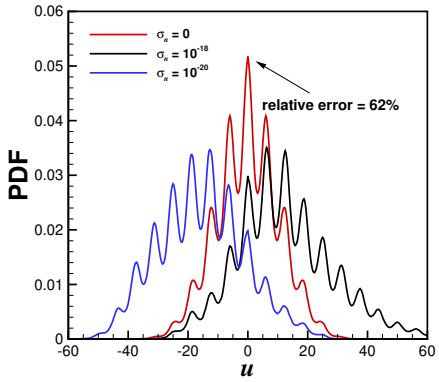

Figure 1: Influence of small disturbances on the probability density functions (PDFs) of ultra-chaos.

Results and Discussions

As shown in Fig. 1, the small disturbances indeed cause the large deviation of chaotic simulations of the damped driven sine-Gordon equation not only in trajectories, but also in statistics. In other words, for the ultra-chaos, statistical results are sensitive to small external disturbances. By varying the physical parameters, it is shown that the so-called ultra-chaos widely exists for the damped driven sine-Gordon equation. Note that, for normal-chaos, although small disturbances lead to large deviation of trajectories, they do not affect the statistics. However, statistical characteristics of an ultra-chaotic system have sensitivity dependance on tiny disturbances. Thus, small disturbances like environmental noises could lead to the loss of the reproducibility of an ultra-chaos. It is hoped that the ultra-chaos could provide us with a new door to study chaos theory.

References

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