Response statistics of a conceptual airfoil with consideration of extreme load conditions

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Abstract. An aircraft sometime serves under extreme flight conditions, which will have a substantial impact on its flight safety. We explore dynamical behaviors of a conceptual airfoil with an extreme random load portrayed by a non-Gaussian Lévy noise. We first theoretically deduce amplitude-frequency equations associated with the deterministic airfoil system. We observe an excellent agreement between the analytical results and the numerical ones, as well as a bistable behavior. Then, the impacts of the extreme random load are numerically examined in depth. Within the bistable regime, the extreme random load can induce stochastic transition and resonance. Interestingly, the Lévy noise is more likely than the Gaussian scenario to cause a highly unexpected stochastic transition. All of the findings would be helpful in ensuring the flight safety and enhancing the structural strength and reliability of aircraft wings operating at extreme flight conditions.

Introduction

The interaction between nonlinearities and stochasticities usually cause sophisticated behaviors than the deterministic systems. Stochastic behaviors of conceptual airfoils with random loads have been extensively investigated [1–5]. In the previous works, we have explored complex dynamics of conceptual airfoil models with Gaussian [1, 2] and narrow-band [3, 4] random excitations via stochastic averaging and multiple-scales methods. But, the previous studies only considered small random fluctuations and effects of extreme load conditions have not been addressed. The idealized Gaussian noise can describe only small fluctuations around the mean value but not large jumps. The non-Gaussian Lévy noise, however, can better model the random loads with both continuous and jumping features [6]. As a result, the purpose of this work is to lead to a better understanding on dynamical behaviors of a conceptual airfoil with extreme random loads modelled as a Lévy noise.

Results and Discussion

The coupled governing equations of the airfoil model with an extreme random load are established as

$$\ddot{h} + \varepsilon^2 x_\theta \ddot{\theta} + \varepsilon^2 \zeta_h \dot{h} + \Omega_h^2 \left(h + \varepsilon^2 \beta_h h^3\right) = -\varepsilon^2 2Q\theta/\mu, \tag{1a}$$

$$\varepsilon^2 \frac{x_\theta}{r_\theta^2} \ddot{h} + \ddot{\theta} + \varepsilon^2 \zeta_\theta \dot{\theta} + \Omega_\theta^2 \left(\theta + \varepsilon^2 \beta_\theta \theta^3\right) = \varepsilon^2 F \sin(\omega t) + \varepsilon \zeta(t), \tag{1b}$$

here θ and h represent the pitch angle and plunge deflection, $0 < \varepsilon \ll 1$ is a small parameter, $Q = [U/(b\omega_\theta)]^2$ is the generalized flow velocity, F and ω are the amplitude and frequency of external force, and $\zeta(t)$ is the extreme random load as a Lévy noise with the stability index α, skewness parameter β, noise intensity D and mean ν. The other symbols can see Ref. [2]. Response statistics of the airfoil systems (1a) and (1b) are shown in Fig. 1. The system parameters are $\mu = 20.0, a = -0.1, b = 1.0, x_\theta = 0.25, r_\theta = \sqrt{0.5}, \bar{\omega} = \sqrt{0.2}, \zeta_h = 0.1, \zeta_\theta = 0.2, \beta_h = 0, \beta_\theta = 0.1, \varepsilon = \sqrt{0.1}$. Bistable behaviors are observed in the airfoil system. The probability $P(\mathcal{A}_{\text{high}})$ gradually decreases as ω increases. Particularly, when $\omega = 1.04$, $P(\mathcal{A}_{\text{high}}) = 0.0191 \ll 1$, thus the undesirable high-amplitude attractor $\mathcal{A}_{\text{high}}$ can be regarded relatively as a rare attractor. The extreme random load can cause a stochastic transition as well as a stochastic resonance. Moreover, a large D or a small α would increase the possibility of stochastic transitions, while the β has basically no effect on them. The Lévy noise is more likely to induce the undesired transitions in comparison with the Gaussian one.

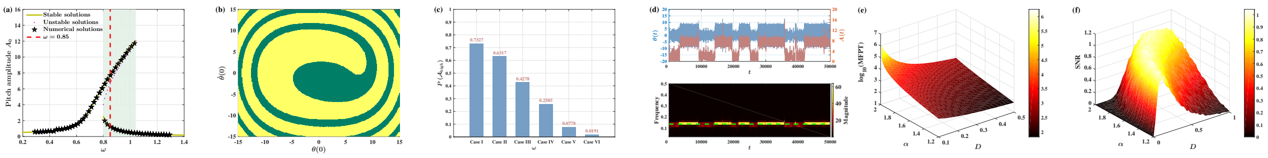

Figure 1: **Response statistics of the conceptual airfoil systems (1a) and (1b) with $Q = 6.0, F = 2.5$**. (**a**) amplitude-frequency curve; (**b**) basin of attraction ($\omega = 0.85$), in which the green and yellow regions respectively corresponds to the low-amplitude attractor $\mathcal{A}_{\text{low}}$ and the high-amplitude one $\mathcal{A}_{\text{high}}$; (**c**) probability $P(\mathcal{A}_{\text{high}})$ for $\omega = 0.83$ (Case I), $\omega = 0.85$ (Case II), $\omega = 0.90$ (Case III), $\omega = 0.95$ (Case IV), $\omega = 1.02$ (Case V) and $\omega = 1.04$ (Case VI); (**d**) time histroy and time-frequency feature ($\omega = 0.85, \alpha = 1.9, \beta = 0, D = 0.2$); (**e**) mean first passage time ($\omega = 0.85, \beta = 0$); (**f**) signal-to-noise ratio ($\omega = 0.85, \beta = 0$).

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