Wave Propagation in Carbon Nanotube with Bilinear Foundation

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Abstract The propagation of harmonic wave in an infinite, single walled carbon nanotube (SWCNT) supported on a bilinear elastic foundation is investigated. The SWCT is modelled as a Euler Bernoulli beam incorporating nonlocal effects invoking using Eringen's stress gradient theory. The foundation stiffness is considered to be disparate in tension and compression, resulting in piecewise linear (PWL) foundation stiffness in the system. Two independent solutions corresponding to the mutually exclusive configurations are considered and impose the matching boundary conditions at the interface. We explore the effect of nonlocality on the realization and stability of traveling wave solutions in such a medium.

Introduction

In recent years, there has been increased interest in carbon nanotubes, their synthesis and mechanics owing to their exceptional mechanical and electrical properties [1]. To name a few, they are being investigated extensively for potential applications as sensors, fibres embedded into matrices, tunable oscillators etc. These studies have modelled these nanoscale systems using atomistic and continuum models. In this study we consider continuum elastic models of SWCNTs. As a result of their ability to undergo large, reversible deformations without developing lattice defects, SWCNTs are assumed to be elastic. There have been extensive studies of SWCNT dynamics using one-dimensional reduced order models such as Euler Bernoulli beam [2], Timoshenko beam and Sanders-Koiter thin shell theory. The nonlocal character of these small-scale systems is incorporated using the stress and strain gradient theory [3]. The SWCNTs are often supported on substrates with nonlinear stiffness and one class of such substrates is an essentially nonlinear PWL stiffness. The effect of PWL stiffness on the propagation characteristics of beams and strings has been studied by Lenci et al. [4]. However, the effect of PWL foundation on wave propagation in SWCNTs with nonlocal effects hasn't been considered previously and is the subject of this study.

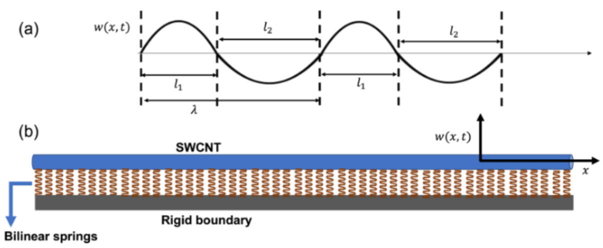

Figure 1: (a) Schematic of one dimensional SWCT, (b) SWCNT supported on PWL spring

$$\rho_0 A w_{tt} + EI w_{4x} - \rho_0 A e_0^2 a^2 w_{ttxx} + k(w) w = 0 \quad (1)$$

$$k(w) = \begin{cases} k_1 \ \forall w \geq 0 \\ k_2 \ \forall w < 0 \end{cases}$$

The equation of motion (Eq. 1) of a SWCNT modelled as a Euler-Bernoulli beam of effective density ρ_0, Young's modulus E supported on PWL foundation (refer Fig. 1) is shown above. We have incorporated the nonlocal parameter e_0 and a is the atomistic characteristic length [4]. The SWCNT is supported on a substrate with PWL stiffness $k(w)$. A single wave consisting of non-identical half wavelengths l_1 and l_2 as shown in Fig. 1(a) is considered. The essential and non-essential boundary conditions at the interface between the configurations $w \geq 0$ and $w < 0$ are matched and additionally, the wave speed for the two configurations is set equal so that the wave propagates without distortion. The resulting set of equations is numerically solved to obtain the half wavelengths, frequency and the constants of integration.

Results

A numerical analysis provides multiple solutions, whereas there are very few physically possible solutions satisfying the conditions. These solutions are numerically continued to explore their evolution as nonlocal and the asymmetry parameter are varied. As a result, stress gradient theory (nonlocality) increases the inertia effect, leading to a reduction in wave speed when compared to the regular beam equation. Asymptotic analysis is performed with the nonlocal parameter as a small parameter and its effect on the realization of periodic solutions and their stability.

References

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