Semi-Implicit Integration and Data-Driven Model Order Reduction for Structures with Hysteresis

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Abstract. Hysteresis models are popular for rate-independent structural damping. Hysteresis in material damping is distributed, nonlinear, and non-analytic. If incorporated in refined finite element (FE) models, numerical time integration is difficult. Generally, implicit integration methods show higher stability for stiff ODE systems arising in FE models. However, integrating the equations of hysteresis using implicit algorithms is difficult. Here we suggest a semi-implicit method where the structural part is treated implicitly and the hysteresis is treated explicitly. Results are very good: time steps can greatly exceed the smallest time period of the structure. Subsequently, we examine model order reduction. Structural modes can be projected directly as usual. Here we propose data driven model order reduction for the hysteretic damping part. The number of internal hysteretic states can be reduced by an order or magnitude with moderate accuracy.

Introduction

The Bouc-Wen model is a popular model for hysteresis. It needs a deformation-related variable that drives the hysteretic response. We formulate a finite element (FE) model of a beam with a lengthwise distributed hysteretic bending moment that is driven by the local curvature. The virtual work integral is estimated using hysteretic states ($\boldsymbol{z}$) that are monitored at Gauss points within each element. The governing equations are

$$\mathbf{M}\,\ddot{\boldsymbol{q}} + \mathbf{C}\,\dot{\boldsymbol{q}} + \mathbf{K}\,\boldsymbol{q} + \mathbf{A}\,\boldsymbol{z} = \boldsymbol{f}_0(t), \tag{1}$$

$$\dot{\boldsymbol{z}} = \left(\bar{A} - \alpha\,\mathrm{sign}\,(\dot{\boldsymbol{\chi}} \circ \boldsymbol{z}) \circ |\boldsymbol{z}|^{n_\mathrm{h}} - \beta\,|\boldsymbol{z}|^{n_\mathrm{h}}\right) \circ \dot{\boldsymbol{\chi}}, \tag{2}$$

where $\boldsymbol{\chi}$ contains the beam curvatures at Gauss points, and the remaining symbols have their usual meanings from structural dynamics and Bouc-Wen hysteresis. The "$\circ$" operator denotes elementwise multiplication; also, the vector absolute value and exponentiation are elementwise as well.

In this paper we address numerical integration of the above FE model under high refinement (hundreds of elements). In our proposed semi-implicit scheme, the structural part is integrated implicitly adapting [1], and the hysteresis part is integrated explicitly while accounting for pointwise within-step rate reversals in the local curvature that drives the hysteretic response. For high n_h or low overall damping, numerical convergence is almost quadratic. For $0.5 < n_\mathrm{h} < 1$ and somewhat larger damping, convergence is approximately linear.
Next, we address modal projection in a refined FE model. Even after modal reduction, there remain very many hysteretic states to be integrated explicitly. Using initial simulation results we identify a subset of hysteretic states that can represent the overall hysteretic behavior. This leads to further model order reduction.

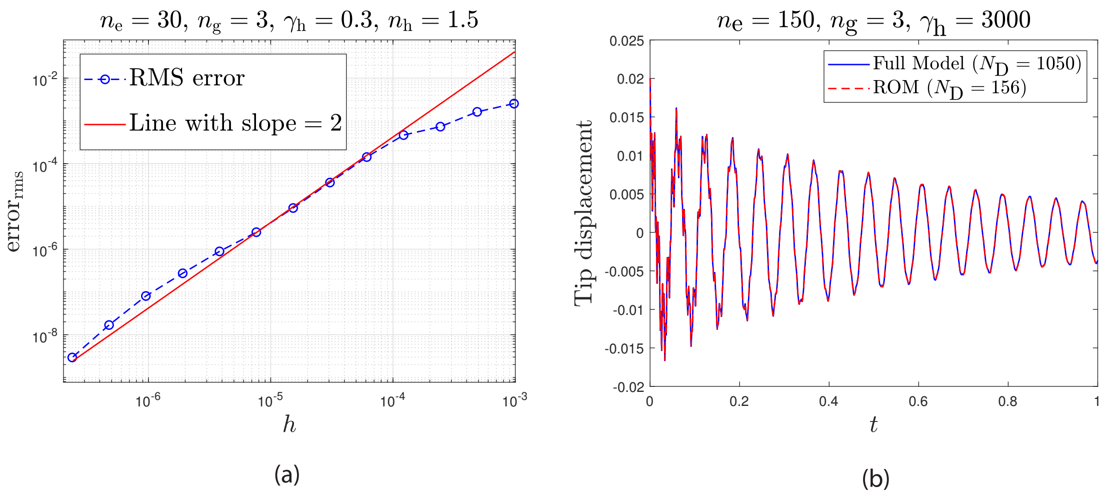

Figure 1: Accuracy of the algorithm and the reduced order model. γ_h is the damping strength, included within matrix $\mathbf{A}$ of Eq. (1).

In Fig. (1(a)), we see nearly quadratic convergence for $n_\mathrm{h} = 1.5$ in an FE model with 30 elements and 3 hysteresis Gauss points in each element. Model order reduction results, starting from a 150 element model, can be seen in a tip displacement plot in Fig. (1(b)).

References

[1] Piché, R., An L-stable Rosenbrock method for step-by-step time integration in structural dynamics. *Computer Methods in Applied Mechanics and Engineering*, 126(3-4): 343-354, (1995).

[2] Bhattacharyya, S., and Cusumano, J. P., An energy closure criterion for model reduction of a kicked Euler–Bernoulli beam. *Journal of Vibration and Acoustics*, 143(4): 041001, (2021).