Spiral organization of *quasiperiodic* shrimp-shaped domains in a delayed discrete predator-prey system

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Abstract.We report on the discovery of quasiperiodic shrimp-shaped structures in bi-parameter space of nonlinear dynamical system. We consider the well-celebrated logistic Lotka-Volterra predator-prey system with delayed harvesting of the prey. The system is discretized by employing the Euler scheme. A detailed investigation of the effort-delay parameter space of the discrete system is carried out based on the Lyapunov exponents and periodicity of the system. We show the existence of periodic and quasiperiodic shrimps in certain parameter spaces. We also reveal that the quasiperiodic shrimp induces torus-bubbling transition to chaos and multistability between different quasiperiodic and chaos dynamics. In certain parameter spaces, the quasiperiodic shrimps are connected along a spiral loop with a Shil'nikov-like quasiperiodic homoclinic connection at the hub of the spiral.

Introduction

Shrimp is a typical regular structure embedded in chaotic regime in the bi-parameter space of nonlinear dynamical systems. It consists of four long narrow antennas connected with a central head. In 1993, Gallas[1] observed the shrimp-shaped periodic domains in the bi-parameter space of the Henon map. Later, the existence of shrimp structure has been found, both theoretically and experimentally, in parameter space of a plethora of nonlinear systems [2-3]. Further, different organizations of shrimps have also been reported. However, researchers mainly focused on periodic shrimp structures (shrimp-shaped domains with periodic dynamics only). Can a shrimp be quasiperiodic? This presentation deals with the discovery of quasiperiodic shrimps and different novel result in dynamics. We consider the logistic Lotka-Volterra predator-prey system with delayed harvesting of the prey as (in normalized form)

$$\left.\begin{aligned} \dot{x}(t) &= x(t)\big(1-x(t)\big) - \alpha\, x(t)y(t) - hx\big(t-\tau\big), \\ \dot{y}(t) &= \alpha\, x(t)y(t) - my(t), \end{aligned}\right\} \tag{1}$$

where τ is the capture delay and $h$is the harvesting effort. Since harvesting is extrinsic in predator-prey dynamics, we explored the global dynamics in effort-delay parameter space after discretizing the system (1) using the forward Euler scheme.

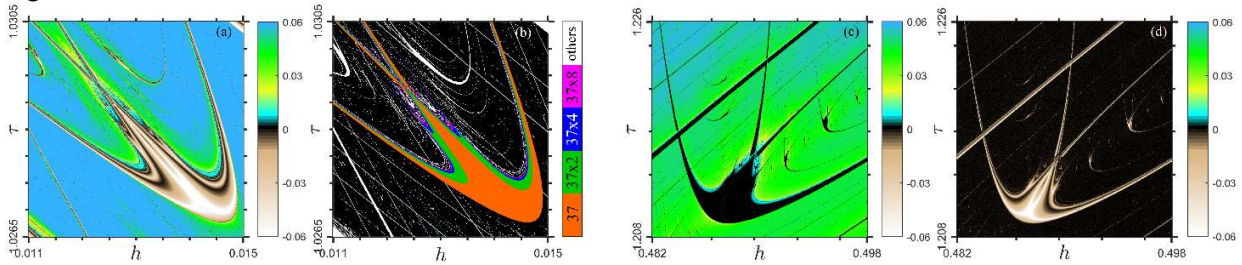

Figure 1: Lyapunov and iso-periodic diagrams showing the existence of (a, b) periodic and (c,d) quasiperiodic shrimps in the effort-delay bi-parameter plane.The color bars in Figs. a, c, and d represent the color code used for the Lyapunov exponents.

Results and discussion

By constructing Lyapunov and iso-periodic diagrams, we have found two types of shrimp-shaped structures, viz., periodic and quasiperiodic shrimps, in the chaotic sea in different (h,τ) parameter plane of the system (cf. Fig. 1).Like periodic shrimp, the quasiperiodic shrimp is formed due to the intersection of two super-stable quasiperiodic windows in the chaotic domain. We also revealed that the existence of the quasiperiodic shrimp implies the existence of torus-bubbling transition to chaos, but the reverse is not true. The system exhibits multi-quasiperiodic, quasiperiodic-chaos, and chaos-chaos multistability near the crossing of the inner antennas of the shrimp. Also, the quasiperiodic shrimps in some effort-delay bi-parameter plane are connected along a continuous spiral loop. At the hub of the spiral structure, the system exhibits a quasiperiodic homoclinic connection to its fixed point. Finally, this discovery could motivate researchers to explore the presence of quasiperiodic shrimps in different dynamical systems, new types of organized connections among the shrimps, as well as experimental demonstration.

References

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