Solution of nonlocal integral problems for hyperbolic equations with discrete effect memory

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Abstract. Nonlocal integral problem for system of hyperbolic equations with discrete effect memory is considered. The memory term contains of the values of unknown function and its partial derivatives at the lines by t. Algorithm for finding solution of nonlocal integral problem for system of hyperbolic equations with discrete effect memory is proposed. The existence and uniqueness of the solution this problem are obtained in the terms of initial data by parametrization method. Results are illustrated by a numerical examples.

Introduction

A partial differential equations with discrete effect memory, i.e. an equation which involves an values of unknown function and its partial derivatives at the some lines, are models many physical problems, for example heat conduction or waves, diffusion processes in chemistry, population evolutions in biology and stability of bridges in architecture. The partial differential equations with discrete effect memory are also called loaded partial differential equations [1]. We study one class of nonlocal integral problems for system of hyperbolic equations with discrete effect memory. For this we use parametrization method [2]. Algorithm of parametrization method is constructed and its convergence is established. Conditions for existence and uniqueness of the solution are obtained in the terms of system's coefficients and boundary matrices. For illustrating results we demonstrate some numerical examples.

Results and Discussion

We consider nonlocal integral problem for system of hyperbolic equations with discrete effect memory in the following form

$$\frac{\partial^2 u}{\partial x \partial t} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + f(t,x) +$$

$$+ \sum_{j=1}^{m} \left\{ D_j(t,x)\frac{\partial u(t_j,x)}{\partial x} + E_j(t,x)\frac{\partial u(t,x)}{\partial t}\Big|_{t=t_j} + G_j(t,x)u(t_j,x) \right\}, \qquad (1.1)$$

$$\sum_{i=0}^{N} L_i(x)u(\theta_i,x) + \sum_{j=1}^{m} \int_{t_j}^{t_{j+1}} K_j(\tau,x)u(\tau,x)d\tau = \varphi(x), \quad x \in [0,\omega], \qquad (1.2)$$

$$u(t,0) = \psi(t), \qquad t \in [0,T], \qquad (1.3)$$

where $u(t,x) = col(u_1(t,x),...,u_n(t,x))$ is unknown function, the $(n \times n)$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, $D_k(t,x)$, $E_k(t,x)$, $G_k(t,x)$, $k = \overline{1,m}$, and the n vector-function $f(t,x)$ are continuous on Ω, $0 \le t_1 < t_2 < ... < t_m \le T$, the $(n \times n)$ matrices $L_i(x)$, $i = \overline{0,N}$, the n vector-function $\varphi(x)$ are continuously differentiable on $[0,\omega]$, $\theta_0 = 0 < \theta_1 < \theta_2 < ... < \theta_N = T$, the $(n \times n)$ matrices $K_j(t,x)$ are continuous and continuously differentiable by x on $[t_j, t_{j+1}] \times [0,\omega]$, the n vector-function $\psi(t)$ is continuously differentiable on $[0,T]$.
By parametrization method the considered problem is reduced to the nonlocal problem for the system of hyperbolic equations with unknown parameters and additional functional relations [3]. Algorithms of finding solution to the obtained problem are constructed, and their convergence is proved. The coefficient conditions for unique solvability of nonlocal integral problem for the system of hyperbolic equations with discrete effect memory are established. Further, we propose an one numerical approach for solve this nonlocal problem. Numerical method based on algorithms of parametrization method [2]. This numerical approach is illustrated by examples [4,5].

References

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