# Model-based Unknown Input Estimation via Partially Observable Markov Decision Processes

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**Abstract**. In the context of condition monitoring for structures and industrial assets, the estimation of unknown inputs, usually referring to acting loads, is of salient importance for guaranteeing safe and performant engineered systems. In this work, we propose a novel method for estimating unknown inputs from measured outputs, particularly for the case of dynamical systems with known or learned dynamics. The objective is to search for those system inputs that will reproduce the actual measured outputs. This can be reformulated as a Partially Observable Markov Decision Process (POMDP) problem and solved with well-established planning algorithms for POMDPs. The proposed method is demonstrated using simulated dynamical systems for structures with known dynamics, as well as a real wind turbine with learned dynamics, which is inferred via use of a Neural Extended Kalman Filter (Neural EKF) scheme, a deep learning-based method for learning stochastic dynamics.

## Introduction

In the domains of Structural Health Monitoring (SHM) and Prognostics and Health Management (PHM), the assessment of performance or condition, e.g in terms of fatigue accumulation and reliability, can be evaluated more efficiently under adequate estimation of the acting loads. One typical such application is input estimation for vehicles (e.g. via estimation of the road roughness profile); a use case which has found increasing use in recent years, as part of the so called on board monitoring or mobile sensing platforms [1].

In this work, we investigate the input estimation problem from a new perspective. The input estimation problem seeks the inputs that reproduce the measured system responses, which are regarded as the actual reference outputs. With such a consideration, the input estimation problem can be formulated as a Partially Observable Markov Decision Process (POMDP). We choose the cross-entropy method [2] for policy search due to its efficiency and robustness.

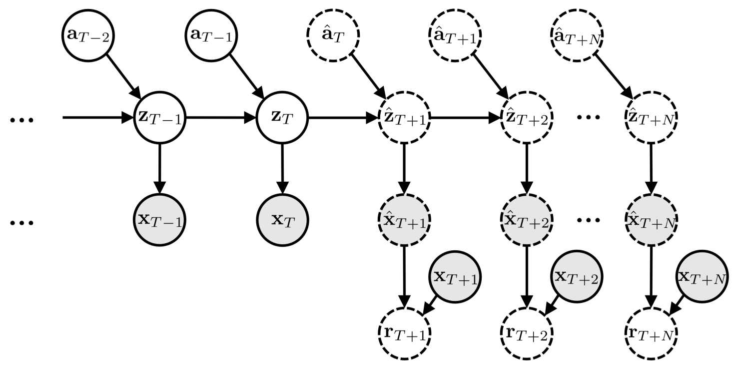

Figure 1: Unknown input estimation problem as a PODMP. The objective is to find a candidate input $\hat{\mathbf{a}}_T$ that can minimize the difference between the observation $\hat{\mathbf{x}}_{T+1}$ that is generated from this candidate input and the true (measured) observation $\mathbf{x}_{T+1}$.

## Results and Discussion

We show the applicability of the proposed methodology in theory and real-world applications. The problem of mobile sensing is examined, where the inputs correspond to road and rail profiles, for the case of road versus railway infrastructure, respectively. These examples demonstrate potential on real-world problems of practical value. Finally, the proposed method is applied on the further problem of input estimation using a real-world dataset obtained from a wind turbine. Here, a deep learning-based dynamics model [3] is first inferred from the available data. Different model-based reinforcement learning frameworks and dynamics modeling methods can be integrated into the proposed methodology. This work aims to set the idea of such a use case for POMDPs in place. The influence of various reinforcement learning methods and a thorough comparison against further input estimation frameworks are left for further work.

## References

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