Reduced order modeling of rotating structures featuring geometric nonlinearity with the direct parametrisation of invariant manifolds method

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Abstract. The direct parametrisation method for invariant manifolds (DPIM) is applied to rotating structures. Reduced-order models of arbitrary order expansion can be derived for non-autonomous systems of nonlinear differential equations stemming from finite element models of continuous structures. The method is applied to a rotating simplified fan blade with comparisons to full order model simulations.

Introduction

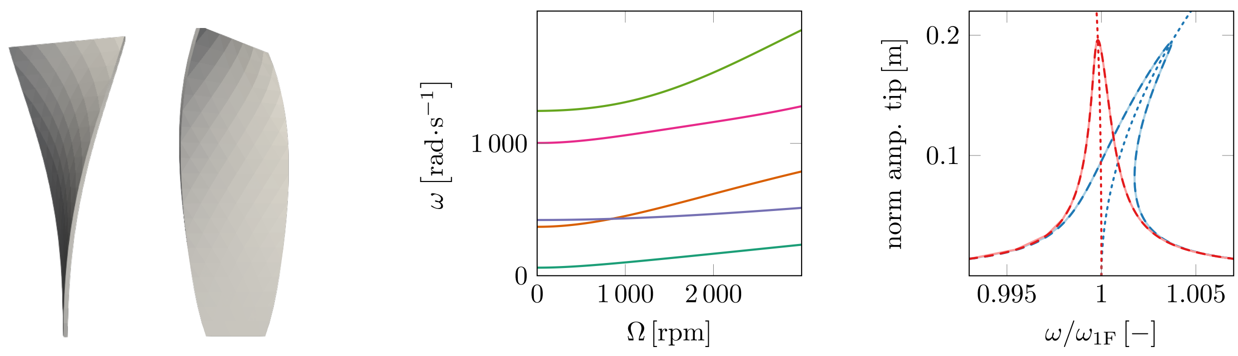
Aircraft engine manufacturers are developing larger systems with longer blades where the geometric nonlinearity is enhanced. As a result, the derivation of efficient reduction method to tackle such problems at the design stage is important.

This work adresses the application of the direct parametrisation method for invariant manifolds (DPIM), following the implementation proposed for non-autonomous problems in [2], to rotating structures subjected to large displacements. This method relies on the assumption that the unknowns of the problem, displacement and velocity, as well as the reduced dynamics equation, can be expressed under the form of arbitrary orders polynomials of the new coordinates of the problem, the *normal coordinates*. Examples of such developments are given for the displacement, eq. (1), and the reduced dynamics, eq. (2). More details are given in a dedicated paper [1].

$$\mathbf{u}(t) = \boldsymbol{\Phi}\mathbf{z} + \sum_{p=2}^{o_a} [\boldsymbol{\psi}(\mathbf{z})]_p + \sum_{p=0}^{o_{na}} \left[\hat{\boldsymbol{\psi}}(\mathbf{z})\right]_p \quad (1) \qquad \dot{\mathbf{z}} = \boldsymbol{\Lambda}\mathbf{z} + \sum_{p=2}^{o_a} [\mathbf{f}(\mathbf{z})]_p + \sum_{p=0}^{o_{na}} \left[\hat{\mathbf{f}}(\mathbf{z})\right]_p \quad (2)$$

Results and discussion

The method is applied to a simplified fan blade, shown in fig. 1a, which is $1\,\mathrm{m}$ long. The Campbell diagram is given in fig. 1b and shows the classic stiffening of the structure with increasing rotation speed. Finally, comparison between full order simulation and ROM computations using a single master mode, are shown in fig. 1c (blue : $0\,\mathrm{rpm}$; red : $2000\,\mathrm{rpm}$). One can observe the perfect match between the reduced model (dashed line) and the full model (solid line) highlighting the efficiency of the method, coupled to a significant time saving in terms of computation (1 day versus 1 minute).



(a) geometry of the blade (b) campbell diagram of the blade (c) comparison of FOM and ROM

Figure 1: Reduced order model of a simple rotating blade;
(b) : —— : 1F; —— : 2F; —— : 1T; —— : 1E; —— : 3F; (c) : —— : FOM; - - - : DPIM $\mathcal{O}(5,4)$; ----- : DPIM $\mathcal{O}(5)$

References

[1] A. Martin, A. Opreni, A. Vizzaccaro, L. Salles, A. Frangi, O. Thomas, and C. Touzé. Reduced order modeling of slender structures subjected to centrifugal effects using the direct parametrisation of invariant manifolds. *in preparation*.

[2] A. Opreni, A. Vizzaccaro, C. Touzé, and A. Frangi. High order direct parametrisation of invariant manifolds for model order reduction of finite element structures: application to generic forcing terms and parametrically excited systems. *Nonlinear Dynamics*, accepted, 2022. doi:10.21203/rs.3.rs-1359763/v1.