Relativistic chaotic scattering

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Abstract. We analyze global properties of chaotic scattering such as the escape time distribution and the decay law of the Hénon-Heiles system in the context of special relativity. Our results show a scaling law between the exponent of the decay law and the β factor is uncovered where a quadratic fitting between them is found. Besides, we study on how time dilation occurs within the scattering region by measuring the time with a clock attached to the particle observing that the several events of time dilation that the particle undergoes exhibit sensitivity to the initial conditions. Finally, we apply these results in the scattering in three-body problem in relativistic regime.

Introduction

Chaotic scattering is a very relevant topic in nonlinear dynamics. Its applications are rooted in numerous fields of physics. There has been little research regarding the influence of the relativistic corrections in chaotic scattering problems. In particular, the global properties of the scattering system as, for example, the escape time distribution and the decay law of the particles, have not been much investigated as in the Newtonian case. Here, it is also studied some relevant characteristics of the exit basin topology of the relativistic Hénon-Heiles system: the uncertainty dimension, the Wada property, and the basin entropy. We also extend the previous results to systems where the gravitational interactions are not negligible once the relevance of the special relativity corrections in the context of chaotic scattering has been highlighted. For this purpose, a simple model which is related to the three-body problem, called the Sitnikov problem is used.

Results and discussion

Firstly, the results show that the average escape time decreases with increasing values of the relativistic factor β. The survival probability of the particles in the scattering region is also studied, uncovering an explicit scaling law between the exponent of the decay law and the β factor [1, 2]. On the other hand, we have used the post-Newtonian approximation for the relativistic Sitnikov problem. The influence of the gravitational radius λ of the primaries in the context of the chaotic scattering phenomena has been considered. Now, the metamorphosis of the KAM islands for which the escape regions change insofar λ increases is shown. Later, the unpredictability of the final state of the system when the gravitational radius changes is highlighted [3].

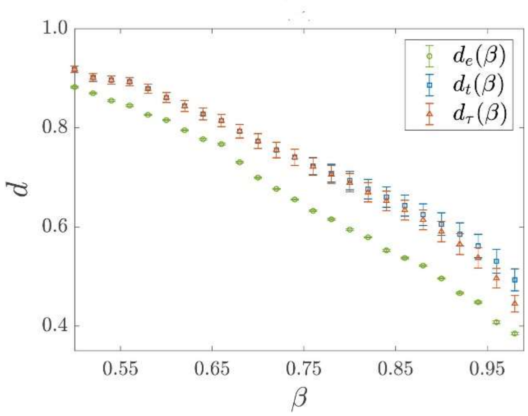

Figure 1: Fractal dimensions of the escape time functions for the cases of inertial (blue) and comoving (red) with the particle frames showing they are relativistic invariants. Green points denote the values of the fractal dimension according to exits.

Finally, in the study of the time dilation by measuring the time with a clock attached to the particle, we observe that the several events of time dilation that the particle undergoes exhibit sensitivity to the initial conditions. However, the structure of the singularities appearing in the escape time function remains invariant under coordinate transformations [4] as shown in Fig. 1 where the values of the fractal dimensions are depicted.

References

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