Uncertainty Quantification in Parameter Estimation Using Physics-integrated Machine Learning

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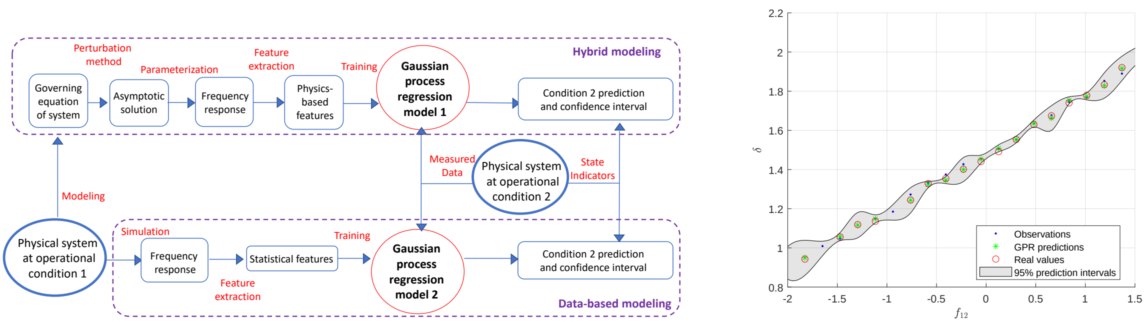
Abstract. This paper proposes a hybrid physics-machine learning modeling method to identify unknown parameters of a nonlinear dynamics system and to quantify the uncertainty of parameter predictions. The contribution of this method depends on the introduction of physics-based features to improve the efficiency of data-based modeling and compensate for the inadequacy of data acquisition by generating training data from parameterization. In the physics-based modeling, the perturbation method is applied to obtain the asymptotic solution and frequency response of the nonlinear system. Extracted mathematical relationships provide for the identification of root causes of changes in frequency response. Subsequently, topological changes are quantified to be used as the inputs of the machine learning model. A Gaussian Process Regression (GPR) model is developed which uses physics-based features. After training and testing, the GPR model fed with measured data from real systems is capable of estimating parameters and the confidence interval of the prediction. In this paper, a coupled duffing oscillator system is simulated as a test system. Then a nonlinear pendulum is employed as the experimental setup to verify our method practically. The accurate predictions of damping and stiffness validated by experimental data demonstrate the effectiveness of the proposed method.

Introduction

Operational conditions of system change due to parametric defects, faults, operational environment and so many inevitable reasons compromising the effectiveness of model-based prediction, analysis and control. Therefore, accurate identification of different operational conditions becomes a very practical problem. However, the complexity of traditional physics-based modeling and the insufficiency in the generalization of data-based modeling leave a lot to be desired in state of the art. This paper is focusing on combining physics-based modeling and data-based modeling to reach higher prediction accuracy and to provide quantified uncertainty of this prediction. The method of multiple scales is leveraged to obtain the frequency response for generating physics-based features. The Gaussian Process Regression (GPR) model is able to incorporate these physics-based features to make predictions and measure the uncertainty over these predictions. The flow chart of the proposed method is demonstrated in Fig. 1 (a).



(a)

(b)

Figure 1: (a) Outline of the proposed method (b) Uncertainty prediction by Gaussian process model

Results and discussion

Gaussian processes (GP) model is a supervised learning method developed by computer science and statistics communities. By integrating prior knowledge as kernel functions, GP provides a probability distribution over possible functions that fit a set of observations. Derived from this, a GPR model can predict the probability distribution of new points, and the corresponding standard deviation can be used to estimate uncertainty over these predictions. Fig. 1 (b) demonstrates the predictions of the coupling coefficient in a system of coupled Duffing oscillators. Trained by four physics-based features (denoted by blue dots), the developed GPR model provides coupling coefficients (denoted by green stars), and 95% confidence intervals (denoted by grey area) with 0.0025 root square error and $6.69e^{-6}$ regression loss, while a data-based GRP model trained by sixteen statistical features including maximum, minimum, median and mean can only reach 0.003 root square error and $8.79e^{-6}$ regression loss. It is clear that the hybrid model outperforms data-based model in accuracy, regression loss and quantity of features. Moreover, these superior results will become more pronounced as the nonlinearity strengthens. The proposed method is also applied to a nonlinear pendulum system for damping and stiffness estimation. The experimental results not only confirm the above conclusions, but also demonstrate the excellent noise resistance ability of the hybrid modeling method.