Algorithmic verification of Lyapunov stability for rigid multi-contact systems subject to impact and friction

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Abstract. Quasi-static object manipulation, grasping, and robotic locomotion tasks require tests of local stability of equilibria involved in the motion plan in order to avoid unpredictable failure. Standard stability tests are not available due to the non-smoothness and discontinuity of rigid body dynamics under unilateral contact and dry friction. In the present work, a recently published algorithmic stability test using semi-definite programming and Lyapunov's direct method is extended. Tailor-made generalizations of Lyapunov's direct method are proposed. The applicability of the extended stability test to multi-contact systems is demonstrated for the first time. As a case study, sufficient stability conditions of a planar rigid body resting on a slope with 2 point contacts are found numerically.

Background

The dynamics of rigid objects and multibody systems under unilateral contact, friction and impact is rich, and only partially understood. Stability analysis of equilibria is particularly challenging. At the same time, verifying the stability of an equilibrium state is essential for many applications like object manipulation, grasping, and robotic locomotion. Lyapunov's direct method is based on constructing a function over state space, which is decreasing along trajectories of the dynamics and has a local minimum point at the equilibrium state under investigation. However the theory does not include a general recipe on how to construct such a function.

It has been pointed out recently [1] that verification of Lyapunov stability by construction of a Lyapunov function can be formulated as an optimization problem over sums-of-squares polynomials, for which efficient numerical algorithms are available. Several examples of Lyapunov stability certificates were presented by [1], but none of those involved multiple contacts.

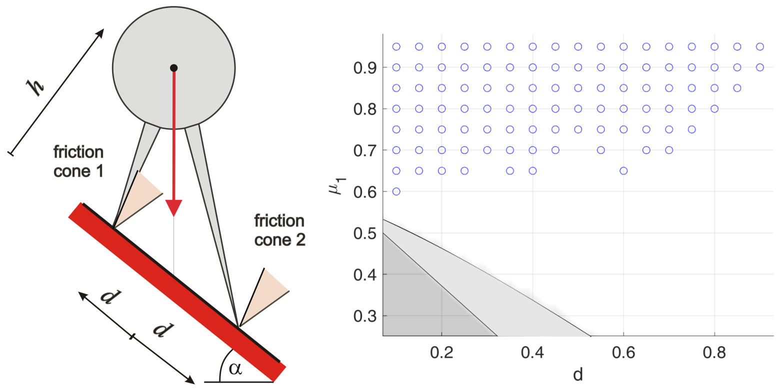

Figure 1: Left: a rigid body on a slope of angle α with 2 point contacts subject to gravity load. The center of the circle is the center of mass, and the radius of gyration of the object is chosen as length unit. Coulomb friction with coefficient μ_i is assumed at both contact points. Right: illustration of the algorithmic stability test for slope angle $\alpha = 30°$; $h = 1$; $\mu_2 = 1$; and inelastic impacts. Background colors show regions of model parameters d, μ_1 where the system is provably unstable (dark grey); unstable according to numerical simulation (light grey) or stable according to numerical simulation (white). Empty circles mark points where the proposed stability test successfully verified stability.

Extensions of Lyapunov's direct method and application to multi-contact equilibria

The verification techniques of [1] were applied by the author to the problem of a planar rigid body with two point contacts on a slope (Fig. 1). Since the original version of the method failed to verify stability in all cases, two extensions of Lyapunov's direct method were proposed. The first one allows the use of piecewise smooth Lyapunov functions constructed as the maximum of several polynomials. The second extension allows Lyapunov functions, which may temporarily increase along trajectories.

The generalized stability test was applied to the same problem. In the case of perfectly inelastic impacts, semi-analytic conditions of stability are available in the literature. In the case of partially elastic impacts, the analytic, exact conditions of stability are not available, but stability can be predicted based on systematic numerical simulations. In both cases, the proposed method could verify stability for many values of model parameters (Fig. 1). To the best knowledge of the author, the proposed method is the first algorithmic Lyapunov stability test successfully applied to multi-contact equilibria.

References

[1] Posa M., Tobenkin M., Tedrake R. (2015) Stability analysis and control of rigid-body systems with impacts and friction. *IEEE Transactions on Automatic Control* **61**: 1423-1437.