A complete stability chart for the tippedisk

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Abstract. The tippedisk is a mathematical-mechanical archetype, showing a non-intuitive inversion behavior, if the disk is spun fast enough. By introducing a full 3D mechanical model and assuming set-valued force laws to account for normal and frictional contact forces, the dynamics of the tippedisk can be studied numerically. In addition, the model dimension can be reduced through model reduction techniques, yielding a lower dimensional model, being perfectly suited for closed form analysis of the qualitative dynamics. Previous work of the authors has shown that the bifurcation scenario contains a heteroclinic bifurcation, followed by a subcritical Hopf bifurcation. In this paper, we derive a complete stability chart that characterizes various bifurcation scenarios in closed form, which allows us to understand the qualitative dynamics of the tippedisk.

Introduction

The tippedisk forms a mathematical-mechanical archetype for friction induced instabilities that shows a counter intuitive inversion phenomenon when spun around an in-plane axis, cf. Fig. 1. As this inversion cannot be explained through energetic arguments, we have to employ to the whole field of nonlinear dynamics to study and understand the behavior of the system qualitatively. Since we aim to study the nonlinear behavior, we seek for closed form expressions that characterize the dynamics of the tippedisk. Due to the singularly perturbed structure of the system equations, tools and methods from singular perturbation and Melnikov theory are applied to derive conditions for the existence of heteroclinic orbits on a slow manifold [2, 3]. Asymptotic analysis is used to obtain closed form expressions.

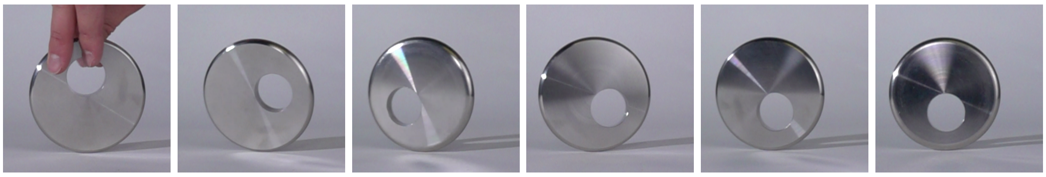

Figure 1: Stroboscopic sequence, showing the inversion phenomenon of the tippedisk.

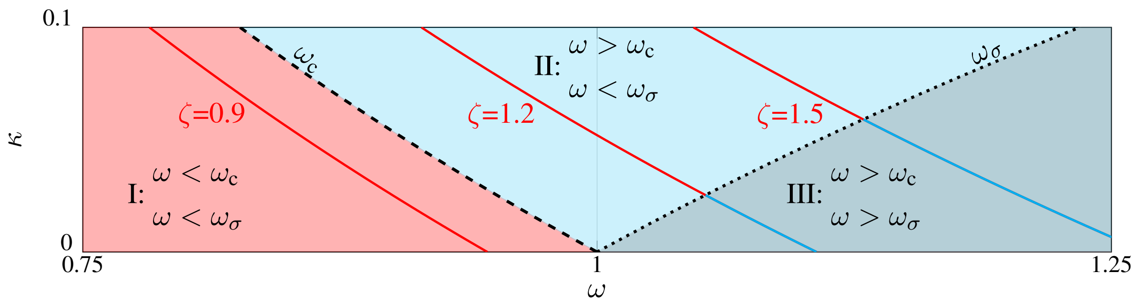

Figure 2: Stability chart: shows different bifurcation scenarios for variants of the tippedisk.

Results and Discussions

Combining the results of [1–3] and introducing dimensionless parameters and system equations, we will derive the complete stability chart Fig. 2 of the tippedisk. This diagram shows stability regions, characterizes Hopf bifurcations as sub- or supercritical, and allows for statements about the existence and asymptotic behavior of hereroclinic saddle connections. Furthermore, the consideration of fundamental dynamic properties provides a complete characterization of the qualitative dynamics of the tippedisk. Various stability regions imply different variants of the tippedisk with qualitatively distinct behavior, but all show the phenomenon of inversion. Validity of the presented analysis is shown by numerically computed bifurcation diagrams.

In summary, the presented analysis explains the inversion phenomenon of the tippedisk and provides a natural, intuitive interpretation of its qualitative behavior from the perspective of nonlinear dynamics.

References

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