Vacuum polarization energy in coupled-fermion ϕ^4 kink systems

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Abstract. We study the quantum effects of kink solitons coupled to a bound fermion. In particular, we compute the fermion vacuum polarization energy that represents the renormalized Dirac sea contribution to the total energy.

Introduction

Boson field theory models in one time and one space dimensions ($D = 1 + 1$) wherein spontaneous symmetry breaking produces discrete, degenerate vacua are almost certain to contain (static) soliton solutions which are characterized by localized energy densities. The field equation for such a soliton is equivalent to minimizing the classical energy, $E_{\rm cl}$. The leading, one-loop quantum correction to $E_{\rm cl}$ is the renormalized sum of the shifts of the zero point energies of the quantum fluctuations. These shifts reflect the polarization of the vacuum induced by the soliton and thus this quantum correction is frequently called the vacuum polarization energy $E_{\rm VPE}$. Then $E_{\rm cl} + E_{\rm VPE}$ is next-to-leading in the semi-classical expansion.

In Refs.[1, 2] new soliton configurations were constructed by coupling the boson to a single fermion bound state and minimizing the quasi-classical energy which is the sum of $E_{\rm cl}$ and this bound state energy E. We have revisited [3] that construction for two reasons. First, in Refs.[1, 2] the Dirac sea contribution was omitted. But in any legitimate expansion scheme this contribution is of the same order as the one from the single level. Second, solutions with the single fermion occupying a negative energy level were also constructed. There is no physical interpretation of such a configuration without the Dirac sea. We therefore consider the fermion part of $E_{\rm VPE}$ which is the renormalized Dirac sea contribution to the energy.

In $D = 1 + 1$ the scalar field Φ is dimensionless while the fermion spinor Ψ has canonical energy dimension $\frac{1}{2}$. To make the Yukawa coupling constant g dimensionless we write the Lagrangian as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{\lambda}{4}\left(\Phi^2 - \frac{M^2}{2\lambda}\right)^2 + {\rm i}\overline{\Psi}\partial\!\!\!/\Psi - g\sqrt{\frac{\lambda}{2}}\,\overline{\Psi}\Phi\Psi\,. \qquad (1)$$

Results and discussion

In all numerical simulations we set the scale by choosing $M = 2$ and eventually vary λ. We have reproduced the kink solitons reported in Ref.[1] who only considered $M^2 = 2\lambda$. In addition we have constructed solutions deviating from this particular case for different values of the Yukawa coupling constant g. Note that $M = 2$ causes g to be the mass of a non-interacting fermion since $\langle\Phi\rangle^2 = \frac{M^2}{2\lambda}$.

With the fermion coupling, the boson soliton profile is a distorted kink but it maintains to be odd under spatial reflection so that there are two parity channels of the fermion spinors. With this separation (the fermion part of) $E_{\rm VPE}$ can straightforwardly be computed using spectral methods [4]. Then the fermion contribution to the total energy has two components: $E + E_{\rm VPE}$. In all cases considered we found that $E_{\rm VPE}$ adds positively. It is substantial and may even outweigh the energy gain from binding the single level so that $E + E_{\rm VPE} \gtrsim g$. This suggests that a consistent treatment of the Dirac sea destabilizes the solutions from Ref.[1]; certainly this is the case when the single level is chosen to be a higher energy excited bound state.

We have also explored the parameter dependence of various energy components (classical, fermion and boson quantum corrections). From this we conjecture that the model is consistent only when $M^2 \gg \lambda$ and that significant binding of the fermions requires $g \sim \frac{M^2}{\lambda} \gg 1$. In that regime the solitons from Ref.[1] may be legitimate solutions.

References

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