Nonholonomic dynamics of steer-free rotor-actuated Twistcar

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Abstract. Underactuated wheeled vehicles are commonly studied as nonholonomic systems with periodic actuation. Two classical examples inspired by riding toys are the Snakeboard and the Twistcar, which were analyzed using planar models. In this work, we present a new model – the steer-free rotor-actuated Twistcar, which combines the characteristics of these two models, and differs from them by having one passive shape variable, and by the added dissipation due to wheels' rolling resistance. Using numerical analysis, we show that the system exhibits multiplicity of periodic solutions. Varying parameters such as actuation frequency and structural length ratio lead to bifurcations, stability transitions, and symmetry breaking of these periodic solutions. We will also present ongoing research progress on experimental demonstration in a robotic prototype, as well as asymptotic analysis of a simplified approximation of the system.

Introduction

The dynamics of underactuated wheeled vehicles governed by nonholonomic constraints have been an extensively researched topic for decades. One of the classic examples is the *Snakeboard* [1], which is actuated by controlling the wheels' heading angles and an oscillating rotor attached to the body. Choosing different gaits of periodic inputs enables steering the Snakeboard along desired paths [1]. Another example is the *Twistcar* [2], in which the joint connecting the body and steering link is periodically actuated, either by prescribing the steering angle or the mechanical torque. Asymptotic analysis of the Twistcar revealed abundant nonlinear phenomena in its dynamics, such as movement direction reversal depending on the vehicle's structure [2]. In both works, all shape variables are actuated, and the body motion shows growing oscillations superposed on the unboundedly growing mean value.

In this work, we introduce the combined model *Steer-free Rotor-actuated Twistcar*, see Fig. 1. It has a single actuation of an oscillating inertial rotor angle $\psi(t)$ relative to the body, while the steering joint angle is *passive*, and evolves dynamically. In addition, we consider viscous dissipation caused by the wheels' rolling resistance, and possibly damping of the passive steering joint. This results in existence of periodic solutions of the system, in which the dissipated mechanical energy per cycle is balanced by the energy input of the actuation.

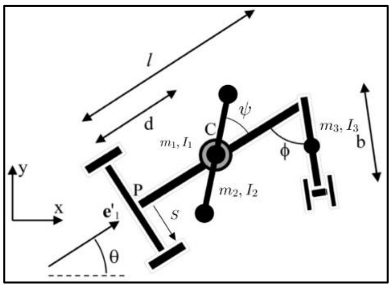

Figure 1: Model of the steer-free rotor-actuated Twistcar

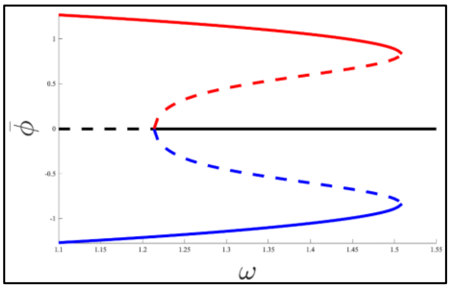

Figure 2: Periodic solution branches – mean steering angle $\bar{\phi}$ as a function of the actuation frequency ω.

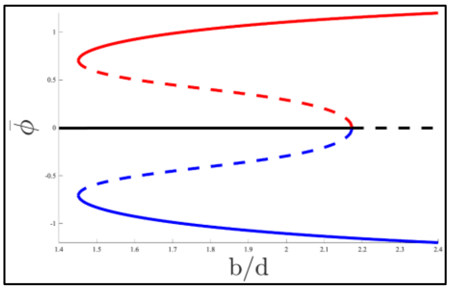

Figure 3: Periodic solution branches – mean steering angle $\bar{\phi}$ as a function of structural length ratio b/d

Results and discussion

Invariance properties of the vehicle's dynamics enable some reduction of the system's dimensionality, and numerical integration is utilized for seeking periodic solutions and analyzing their orbital stability via the evaluation of Floquet multipliers. We find multiplicity of periodic solutions, some are stable (i.e. convergent from nearby perturbed initial states) while others are unstable (i.e. divergent). Continuously varying a single parameter of the system while tracking all branches of periodic solutions reveals interesting bifurcations and stability transitions. Fig. 2 and Fig. 3 show solution branches of the mean value of the passive joint angle $\bar{\phi}$ as a function of the actuation frequency (Fig. 2) and structural length ratio b/d (Fig. 3). The solid curves denote stable periodic solutions whereas the dashed curves denote unstable ones. One can see that the symmetric periodic solution $\bar{\phi} = 0$, having zero net body rotation, changes from unstable to stable through a subcritical pitchfork bifurcation where a pair of unstable asymmetric branches evolve. Another critical transition is folded (saddle-node) bifurcation where the unstable asymmetric solution branches fold back into stable asymmetric branches. Remarkably, this implies that for some regions of the parameters (ω or b/d), the only stable periodic solution is symmetric, while in another region the only stable solutions are pair of asymmetric ones, and in an intermediate region the systems exhibit multi-stability of symmetric and asymmetric solutions. Finally, we plan to present ongoing research progress on experimental demonstration in a robotic prototype, as well as asymptotic analysis of a simplified approximation of the system, in the spirit of [2].

References

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[2] Chakon, O., & Or, Y. (2017). Analysis of underactuated dynamic locomotion systems using perturbation expansion: the twistcar toy example. *Journal of Nonlinear Science*, *27*(4), 1215-1234.