Response statistics of a conceptual two-dimensional airfoil in hypersonic flows with random perturbations

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Abstract. We investigate a conceptual pitch-plunge airfoil in hypersonic flows, considering the effects of irregular fluctuations and external random load modeled as a Gaussian white noise. The dynamical responses of the airfoil model especially the bifurcation behaviors are studied via the harmonic balance method. Subsequently, the effects of stochasticity on the hypersonic airfoil system are explored in the regimes of subcritical and supercritical Hopf bifurcations depending on the system parameters. Several interesting phenomena, in particular, intermittency and stochastic transition between the low-amplitude oscillation state and the undesirable high-amplitude oscillation state are triggered under random perturbations. This work will provide new insights into the safety and reability design of hypersonic aircraft.

Introduction

Hypersonic airfoils generally refer to the ones at Mach number greater than five and the aerodynamic loads are greater. The existence of nonlinearities makes the airfoil system undergo Hopf bifurcation [1,2]. Moreover, the flight environment of aircraft is quite complex, which contains many stochastic factors including irregular fluctuations and external random load [3-5]. However, there is less research on the stochastic dynamics of airfoil models in hypersonic flow. The influences of stochastic disturbance on the hypersonic airfoils are not sufficiently studied. We study the stochastic response of a conceptual two-dimensional airfoil excited by irregular fluctuations in the flow and external random load modeled as a Gaussian white noise.

Results and Discussion

The dimensionless coupled motion equations with the external random load are established as

$$
\begin{aligned}
\xi'' + x_\alpha \alpha'' + 2\zeta_\xi \frac{\varpi}{U}\xi' + \left(\frac{\varpi}{U}\right)^2 G(\xi) &= O_{EA}, \\
\frac{x_\alpha}{r_\alpha^2}\xi'' + \alpha'' + 2\zeta_\alpha \frac{1}{U}\alpha' + \frac{1}{U^2}M(\alpha) &= P_{EA} + \eta(t),
\end{aligned}
$$

in which ξ and α denotes the dimensionless plunge and pitch degrees of freedom, ϖ is the intrinsic frequency ratio, r_α is the radius of gyration about the elastic axis, ζ_ξ, ζ_α are the damping coefficients of the plunge and pitch degrees of freedom, and O_{EA}, P_{EA} are the dimensionless aerodynamical coefficients. The external random load $\eta(\tau)$ satisfies $E[\eta(t)] = 0, E[\eta(t)\eta(t+\tau)] = D\delta(\tau)$, where D is the noise intensity. The irregular fluctuations in the flow can be written as $U(t) = U_m + \phi(t)$, where U_m is the average flow velocity and $\phi(t)$ is the fluctuating component with zero mean. The obtained results are shown in Fig.1. Bistable behaviors are observed and the effects of the system parameters are discussed. Under the random loads, stochastic transitions and stochastic P-bifurcations are found in both supercritical and subcritical bifurcation region.

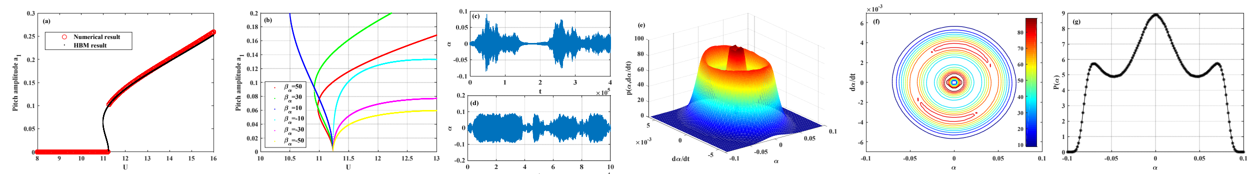

Figure 1: Response statistics of the conceptual airfoil systems. (a) Pitch amplitude versus the flow velocity U; (b) The subcritical and supercritical region with different β_α; (c) Time history of pitch motion in supercritical region with $U_m = 13.88$; (d) Time history of pitch motion in subcritical region with $D = 1 \times 10^{-10}$; (e,f) Steady-state joint probability density function and countours with $U_m = 13.95$; (g) Steady-state marginal probability density function with $D = 1 \times 10^{-10}$.

References

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