Closed-form solutions and conservation laws for a Korteweg-de Vries-like equation

Chaudry Masood khalique*, and Mduduzi Yolane Thabo Lephoko

**International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X 2046, Mmabatho 2735, Republic of South Africa, ORCID # 0000-0002-1986-4859*

Abstract. In this talk we study a second-order nonlinear Korteweg-de Vries-like (KdV-like) partial differential equation, which has several applications in various scientific fields. Firstly, we compute Lie point symmetries of the KdV-like equation. Thereafter the commutator table for the Lie point symmetries is generated, and we use Lie equations to produce one-parameter groups of transformations. Moreover, group-invariant solutions are obtained under each point symmetry. Furthermore, we use the conservation theorem due to Ibragimov to derive the conservation laws for the KdV-like equation.

Introduction

In [1] the authors introduced the new KdV-like equation

$$\left(D_{3,x}D_{3,t} + cD_{3,x}^2 + D_{3,x}^4\right) F \cdot F = 2F_{xt}F - 2F_tF_x + c\left(2F_{xx}F - 2F_x^2\right) + 6F_{xx}^2 = 0 \tag{1}$$

using the generalized bilinear derivative with the prime number $p = 3$. Three classes of different rational solutions were obtained using rational polynomial functions. Moreover, 2 and 3D plots of the rational solutions were presented in [1]. The authors of [2] studied the generalized bilinear differential equation of the KdV-like

$$\left(D_{3,x}D_{3,t} + D_{3,x}^4\right) f \cdot f = 2f_{xt}f - 2f_tf_x + 6f_{xx}^2 = 0 \tag{2}$$

and derived two classes of rational solutions to the resulting KdV-like equation. We note that when $c = 0$ in (1), we obtain equation (2). In this talk we study equation (2) from the symmetry standpoint.

Results and discussion

We start by constructing the Lie point symmetries of KdV-like equation (2) by using the Lie invariance criteria [3, 4]. This gives us four dimensional Lie algebra L_4 spanned by the four point symmetries. We then construct the commutator table for these four symmetries and on invoking Lie equations we obtain one-parameter groups of transformations. Thereafter, we perform symmetry reductions and derive group invariant solutions under each point symmetry. One such solution is the exponential function given as

$$u = e^{-\frac{1}{2}\varepsilon t^2}(C_1 x + C_2), \tag{3}$$

where ε, C_1 and C_2 are constants. In order to understand the physical meaning of the obtained solutions we present two and three dimensional plots.

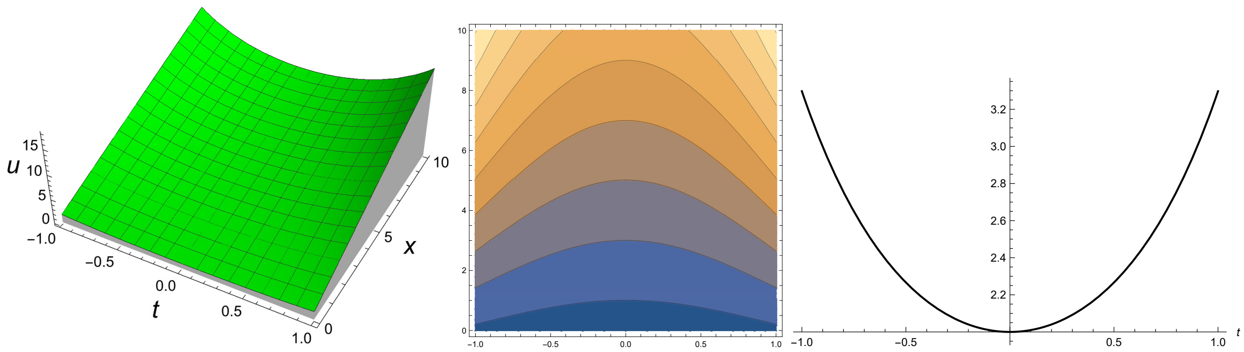

Figure 1: The dynamics of the group invariant solution (3) at $\varepsilon = C_1 = C_2 = 1$, furnishes a parabolic wave structure given in two and three dimensions. This wave structure has a significant application in electrical and electronics engineering. A parabolic antenna depicts an antenna that utilizes a curved surface parabolic reflector that has a cross-sectional shape. This parabola shape enables it to easily direct the radio waves. Besides, the most frequent used is a dish antenna or parabolic dish which has the shape of a dish. The main benefit of a parabolic antenna is the fact that it has high directivity [5].

In addition, the conserved quantities associated with the four symmetries are calculated using Ibragimov's theorem [6]. It is well known that the conserved quantities are very pertinent in physical sciences owing to their robust applications.

References

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