Time-dependent stability margin for autonomous, piece-wise, and discontinuous system

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Abstract. In recent years, a variety of new metrics to analyze dynamical systems emerged. One of them is a metric called time-dependent stability margin, introduced for the first time in [1]. It is used for systems with coexisting stable attractors. The metric evaluates the stability of the system along the periodic orbit, indicating its reliability and resistance to perturbations. In this paper, the time-dependent stability margin is applied to investigate the real-world, discontinuous, and piece-wise mechanical system with impacts. Numerical results show that the periodic orbit vulnerability to perturbations is increased in close vicinity to the particular impact events. It may induce implications because after time impacts wear the system causing plastic deformation or backlash in the mechanisms. Consequently, it may lead to perturbation sufficient enough to change the attractor.

Introduction

Multi-stable systems are commonly known in mechanical engineering, mathematical biology, fluid dynamics, control engineering, and others. Over the years, a variety of techniques to analyze such systems were developed. Among others, we can distinguish basin stability metrics, basin entropy, survivability, and time-dependent stability margin.

The time-dependent stability margin [1] is a method designed to examine the stability of the system along the periodic orbit. In multi-stable mechanical systems due to coexisting attractors, we can experience a sudden change in dynamical response. The phenomena may be severe for the system as it can trigger undesired behavior. It can also be used to facilitate control over the system. The cost of control can be minimized by applying the impulse at the appropriate time when the stability margin of the current attractor is the smallest. Therefore, such analysis can provide significant advantages when a mechanical system and its control are designed.

In order to test the usefulness of the metric in real-world applications, we perform a time-dependent stability analysis of the novel-yoke-bell clapper system (Figure 1a) with variable geometry. The system is piece-wise due to the nature of excitation and discontinuous due to impacts between the bell and the clapper. The mathematical model of the system was experimentally validated in [2].

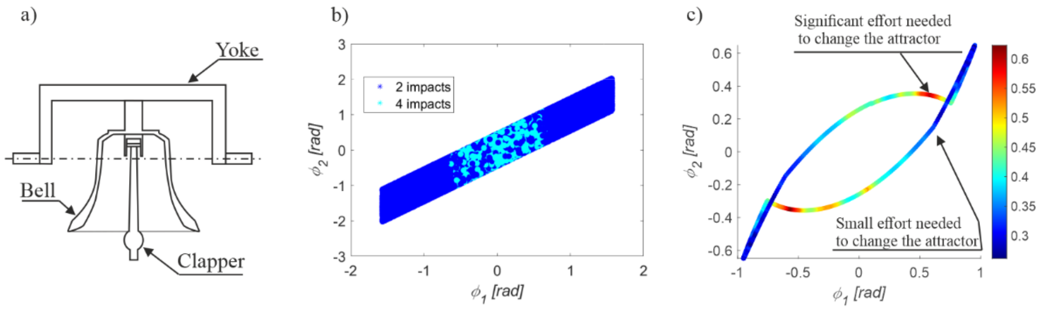

Figure 1: a) Schematic view of the considered system b) Basin of attraction c) Evolution of the stability margin along the attractor with four impacts per period of motion.

Results and discussion

Detailed information considering how the metric is calculated is described in [1]. Figure 1b shows two coexisting attractors (two and four impacts per one period of motion) in phase space, where ϕ_1 is the angular displacement of the bell and ϕ_2 is the angular displacement of the clapper. The stability margin along the attractor with four impacts per period of motion is presented in figure 1c. The metric indicates parts of the orbit that are the most prone to perturbations. In this case, it is in close vicinity to the first and third impact event. On the other hand, the safety margin is the biggest before the second and fourth impact.

After time, impacts wear the structure causing plastic deformation or backlash in the mechanism. As a consequence, the trajectory of the bell or the clapper may be changed and a disturbance may be introduced to the system possibly leading to a sudden change of attractor. This study shows the practical application of the novel tool for the analysis of multi-stable systems. It allows for further investigations considering the control of the system (by means of the desired transition between attractors) or optimization for robustness and safety.

References

[1] Brzeski, P. et. al. (2018) Time dependent stability margin in multistable systems. Chaos: An Interdisciplinary Journal of Nonlinear Science. *Chaos* **28**: 093104.

[2] Burzyński, T, et. al. (2022) Dynamics loading by swinging bells – Experimental and numerical investigation of the novel yoke-bell-clapper system with variable geometry. *Mechanical Systems and Signal Processing* **180**:109429.