## Coexistence of hidden attractor and self-excited attractors on the plane

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**Abstract**. In this work, we introduce a class of continuous time planar systems that presents the coexistence of hidden and self-excited attractors. This class of planar systems is derived from three-dimensional piecewise linear (PWL) systems. Then, we present an approach to generate self-excited and hidden multiscroll attractors by defining a vector field on  $R^2$  with an even number of equilibria.

## Introduction

There are three classes of attractors, the first class is given by those classical attractors excited from unstable equilibria called *self-excited attractors* whose basin of attraction intersects at least a neighborhood of an equilibrium point, [1] and they are not difficult to find via numerical methods, and the second class is called *hidden attractors* whose basin of attraction does not contain neighborhoods of equilibria. The coexistence of a self-excited attractor and a hidden attractor was observed in the Chua's circuit and reported in [2]. The last class is generated by vector fields without equilibria called *non-self-excited attractors* [3]. Notice that a non-self-excite attractor is given by a system without equilibria, therefore satisfies the definition of hidden attractors. Multistability is the coexistence of two or more attractors, different scenarios of multistability have been reported in [4, 5]. Recently, a class of continuous time dynamical planar systems which was generated by means of the use of hysteresis and at least two unstable focus [6]. This class of systems shows stretching and folding behavior due to hysteresis.

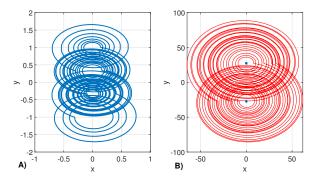


Figure 1: A) Four-scroll attractor. B) Coexistence of two self-excited attractors in blue and a hidden attractor in red.

## **Results and discussion**

In the same spirit that [6], we derive a class of planar systems through a three-dimensional piecewise linear (PWL) systems that have two manifolds, one stable and the other unstable, to generate heteroclinic chaos. Then, we present an approach to generate self-excited and hidden multiscroll attractors by defining a vector field on  $R^2$  with an even number of equilibria. The vector field is defined by affine linear systems such that each equilibrium point is a unstable focus point. So the space is partitioned in hyperbolic set. We start by generating a self-excited multiscroll attractor based on heteroclinic orbits. Interesting phenomena appear when the equilibria are separated by pairs, firstly, the system presents only one basin of attraction which is divided accordingly with the separation of the equilibria, and the coexistence of different self-excited double-scroll attractors arise. At a certain separation of equilibria, a hidden multiscroll attractors emerges, see figure 1.

## References

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