Analysis and bifurcations of non-smooth Filippov predator prey system with harvesting

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Abstract. In this study, it is assumed that harvesting is permitted only when the ratio of prey to predator is below a threshold value. The Filippov type dynamical system consisting of two smooth subsystems is investigated. The existence of harvesting factor within an interval dependent on biologically parameters of the system would ensure the existence and stability of interior equilibrium state for the harvested system. The boundary that splits the two subsystems is classified into sliding and crossing region. It is also proved that there does not exist any escaping region on the boundary. Various equilibrium points such as boundary points, tangent points, pseudo-equilibrium points are found out for the boundary. The visibility condition for these tangent points are determined. Touching bifurcation and boundary bifurcation of the Filippov system is demonstrated numerically. The half saturation rate of predators plays a crucial role in determining whether the system to lie in specific sub-regions or is oscillating in between the two regions.

Introduction

In this study, the Filippov predator prey is formulated which balances the conflict between the exploitation and coexistence of both the species in the environment. When enough food (prey) is available to the predators, there is no hindrance in the coexistence of both the species. However, when the prey to predator availability is below a threshold value, the pressure on predators to survive increases. This pressure can either be decreased by providing additional food to the predators or by harvesting the predators. With the assumption that the predators are of economic interest. So, harvesting of predators is adopted as a control measure. It is assumed that harvesting is permitted only when the ratio of prey to predator is below a threshold value. Whenever the ratio is above this threshold value, harvesting is not allowed. The predator-prey system with the aim of controlling the predator population to level that conserve preys is considered. The hypothesis involved is that human need for harvesting increases with predator abundance and decreases with predators exiguous. This is also due to the fact that there is a threshold above which the financial damage is sufficient to justify the measure. From the analysis of this Filippov system it is concluded that the coexistence of the species is possible in two ways- either the existence of interior equilibrium states or the existence of limit cycle about the interior equilibrium state and the half saturation rate of predators plays a crucial role in determining this dynamical behaviour of the Filippov system.

Recently, application of Filippov systems has been extended to ecological models. Filippov epidemic model was discussed in [1, 2, 3]. Liu proposed a prey-dependent consumption model with impulsive control strategy [4]. The existence of a globally stable pest-eradication periodic solution when the impulsive period is less than some critical values is proved. The dynamics of a non-smooth predator prey system characterized by density-dependent intermittent refuge protection of the prey was studied by Bhattacharyya and Chattopadhyay [5]. When the level of apprehension of prey species is less than a threshold value, the existence of pair of pseudo equilibrium was established.

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