# Regularity and spatially distributed solutions for interacting gases in complex domains

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**Abstract**. Modeling the interaction of gases in complex geometries is typically a hard work due to the difficulty in predicting what kind of interaction is relevant given by diffusion, advection or reactive/absorptive mechanisms. This is particularly important in mixtures of gases, where the interaction can lead to an explosive condition. For instance in the fuel tanks. Our intention is to introduce a model for the interaction between oxygen and nitrogen, develop new regularity conditions and obtain flat solutions together with spatially distributed ones.

## Introduction

The interaction between gases can be described based on diffusion, advection and reaction/absorption. In the presented work, we are concerned with the possibility of having an explosive mixture of gases leading to hazardous consequences (see the Boeing 747-131 that crashed over the Atlantic Ocean in 1996). The aerospace authorities claimed for a solution making an inerted atmosphere introducing nitrogen in the fuel tank (see [1] and the models [2] and [3]). The model in [3] was solved by the travelling waves, but no results were provided about the regularity of solutions or other kinds of solution: flat solutions and spatially distributed.

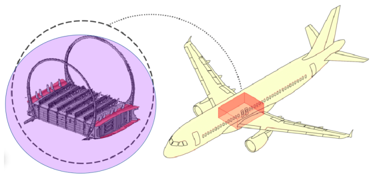

Figure 1: A320 Centre Wing Box. The purple sphere has a radium of $6.249m$. The volume $|x| \leq 6.249m$ was obtained as a result (see the next section) and covers the center tank. The oxygen concentration is at a safe level $\Theta = 0.0258$ at any location within the volume.

## Results and discussions

The model to describe the interaction between the nitrogen ($N$) and the oxygen ($\Theta$), is ([3]): $N_t = \rho \Delta N + a \cdot \nabla N + \Theta^n$; $\Theta_t = \sigma \Delta \Theta + a \cdot \nabla \Theta - N^m$ ; $\Theta_0(x), N_0(x) \in L^\infty(\mathbb{R}^3) \cap L^1_{loc}(\mathbb{R}^3)$, $\rho$ and $\sigma$ are the gases diffusivities, $a$ refers to the vector of vented convection and $n, m \in (0,1)$ are two constants to be calibrated with a real scenario. Given the parabolicity of the involved operator, and considering positive initial distributions in the domain, it holds that $(N, \Theta) \in C^{2+\varepsilon,\, 1+\varepsilon/2}((0,T) \times \mathbb{R}^3)$, $\varepsilon > 0$, together with $\Theta_t \leq 0$ and $N_t \geq 0$. The study of regular and flat solutions follows from a change of variable and the comparison with previously known results. Indeed making: $N(x,t) = N(\eta(x,t),t), \Theta(x,t) = V(\eta(x,t),t), \eta = x + at$, the system is converted into a problem whose dynamics acts in the advection vector and has the form: $N_t = \Delta_\eta N + \Theta^n, \Theta_t = \Delta_\eta \Theta - N^m \leq \Delta_\eta \Theta + N^m$. This last problem has been shown to be regular in [4] leading to conclude that the solutions to our problem are sufficiently regular and do not exhibit blow up in $(0,T)$. Based on this, it is possible to obtain two flat bounding solutions given by the reaction/absorption terms: $\hat{N}(t) = At^{\frac{1+n}{1-mn}}, A^{1-mn} = \frac{(1-mn)^{1+n}}{(n+1)(m+1)^n}, \hat{\Theta}(t) = Bt^{\frac{1+m}{1-mn}}, B = \frac{A^m(1-mn)}{1+m}$. In the search of spatially distributed solutions we make use of selfsimmilar profiles of solutions. This approach is valid in the case of slow advection (this is the case as $a = 0.0125\, m/min$, see [3]) so that the selfsimilar scaled symmetry is not much impacted. The solution for the oxygen is:

$$\Theta(x,t) = \Theta_0(x) - A_\Theta \int_{R^3} (|x-r|)^{\frac{1+m}{1-mn}-2} B_\Theta(t) e^{-\frac{|r|^2}{4t}} dr, \qquad (1)$$

where $A_N = \frac{1}{2\Gamma(2\alpha_N+1)}, A_\Theta = \frac{1}{2\Gamma(2\alpha_\Theta-1)}$, $\Gamma$ is the Gamma function. $B_N(t) = t^{-\alpha_N}, B_\Theta(t) = t^{1-\alpha_\Theta}$ with $\alpha_N = \frac{1}{2}\frac{2-(n-1)m}{1-mn}, \alpha_\Theta = \frac{1}{2}\frac{1+m}{1-mn}$. For particular values in $n, m$ refer to [3]. According to the data in [1] for a single filter working, at $t = 60\, min$, the level of oxygen concentration is $\Theta = 0.0258$. According to the solution in the expression (1), it holds that: $0.0258 \geq 0.2 - A_\Theta |x|^{\frac{1+m}{1-mn}-2} B_\Theta(t = 60\, min) e^{-\frac{|r|^2}{4 \cdot 60}} V_T$. A solution for this last expression is $|x| \leq 6.249m$ (see the Figure 1).

## References

[1] Bahrami, A. (2008) Fuel Tank Ignition Source Prevention Guidelines. FAA Advisory Circular.

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[3] Palencia, J.L.D. (2021) Travelling Waves Approach in a Parabolic Coupled System for Modelling the Behaviour of Substances in a Fuel Tank. *Appl. Sci.*, **11**, 5846.

[4] Escobedo, M. and Herrero, M. (1991) A uniqueness result for a semilinear reaction-diffusion system. *Proc. Amer. Math. Soc.*, **112**, 175-185.