Stochastic analysis of a bistable piezoelectric energy harvester with a matched electrical load

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Abstract. We present an analysis of a bistable piezoelectric energy harvester subject to random mechanical vibrations and with improved performance thanks to the use of a matching electrical network, that optimizes the energy transfer to the electrical load. The model exploits a stochastic differential equation describing the harvester, matching network and load dynamical system. Analytical methods and different numerical techniques are used for its solutions. Results show that, even for the case of random mechanical vibrations, the application of the matching network improves the performances by a significant amount.

Introduction

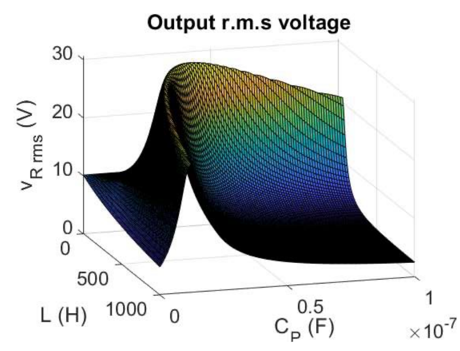
One of the main performance limitations for a piezoelectric energy harvester is the sub-optimal energy transfer from the mechanical source to the electrical load, a condition that can be conveniently represented as an impedance mismatch between the electrical equivalent of the entire electro-mechanical system and the load. This suggests to interpose a proper matching network between the harvester and the load to eliminate the mismatch [1,2,3].

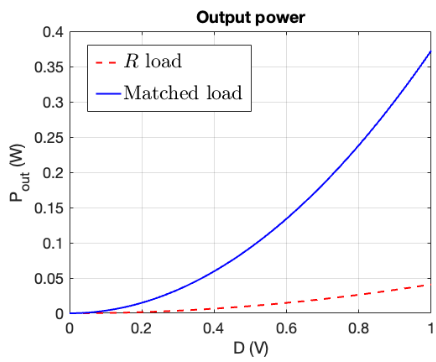
In the simplest case of purely sinusoidal vibrations, i.e. when their energy is concentrated at a single frequency, a relatively straightforward analysis of the harvester is possible [1]. However, a more physically sound description considers the vibration energy spreading on a relatively wide frequency spectrum, thus requiring the use of a stochastic process that, for a negligible noise correlation, can be conveniently modelled with a white Gaussian noise forcing term.

In this contribution, we model a bistable piezoelectric energy harvester subject to random mechanical vibrations, and present novel results through analytical and numerical analysis. The mathematical model is derived from the properties of the mechanical part, from the constitutive equations of linear piezoelectric materials, and from the circuit description of the electrical load. The model includes nonlinearities in the mechanical elastic potential. The equations of motion are stochastic differential equations, here solved using various perturbation methods and different numerical integration schemes. Inspired by our recent work on the application of circuit theory to improve the efficiency of energy harvesting systems, we apply a *LC* matching network to the load [1,3], and we assess the advantage offered by the modified load in terms of output average voltage, output average power and power efficiency.

Results and discussion

We have performed Monte Carlo simulations for the bistable energy harvester with and without (resistive load) the *LC* matching network. The SDEs have been solved numerically using different numerical integration schemes, including Euler-Maruyama, strong order 1 stochastic Runge-Kutta, and weak order 2 stochastic Runge-Kutta [4]. The figure below shows on the left, the output voltage rms value for the harvester with matched load, versus the values of the matching
network parameters *L* and *C*. Optimal values of the parameters maximizing the harvested voltage are clearly recognizable. The right part shows a comparison of the average harvested power by the harvester with resistive and matched load, versus the noise intensity. Optimum values of parameters of the matching network were chosen. The matched solution offers about nine times more power with respect to the simple resistive load.





Left: Root mean square value for the output voltage vs. the *LC* matching network parameters. **Right:** Comparison of the average harvested power for the harvester with resistive load and with matched load, versus the noise intensity.

References

[1] M. Bonnin et al., Nonlinear Dynamics, vol. 104, no. 1, pp. 367–382, 2021.
[2] D. Huang et al., Nonlinear Dynamics, vol. 97, no. 1, pp. 663–677, 2019.
[3] M. Bonnin et al., Energies, vol. 15, no. 8, p. 2764, 2022.
[4] S. Särkkä et al., *Applied stochastic differential equations*. Cambridge University Press, 2019.