Model reduction of a periodically forced slow-fast continuous piecewise linear system

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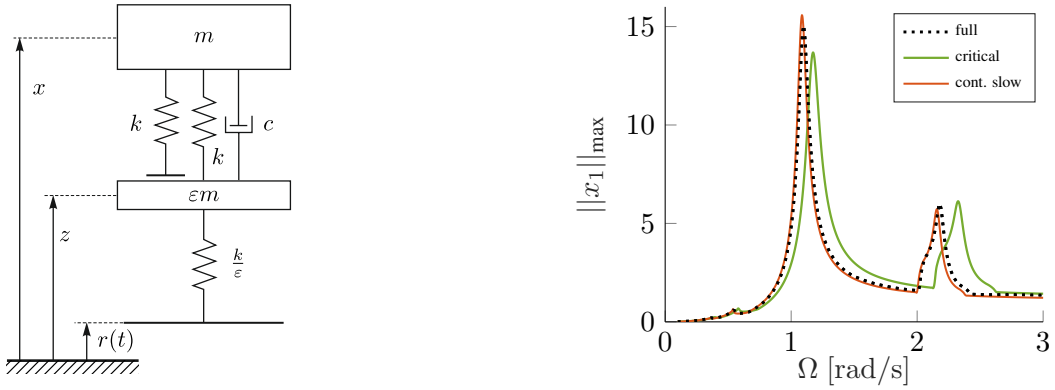
Abstract. In this work, singular perturbation theory is exploited to obtain a reduced order model of a slow-fast piecewise linear 2-DOF oscillator subjected to harmonic excitation. The nonsmoothness of piecewise linear nature is studied in the case of bilinear damping as well as with bilinear stiffness characteristics. We propose a continuous matching of the locally invariant slow manifolds obtained in each subregion of the state space, which yields a 1-DOF reduced order model of the same nature as the full dynamics. The frequency response curves obtained from the full system and the reduced models show that the proposed reduction method can capture nonlinear behaviors such as super- and subharmonic resonances.

Introduction

Invariant manifolds play a major role in understanding the behavior of nonlinear dynamical systems. Among their properties, such manifolds can be used to obtain a reduced dynamics capturing the main features of the original system. The existing methods to find these manifolds often require smoothness properties of the system. A typical example is the theory of singular perturbations, where the reduction to a smooth slow manifold yields a reduced order model describing the slow dynamics of the original system. However, this theory cannot be applied on systems containing nonsmooth nonlinearities without suitable extension to take the nonsmoothness into account. A prominent class of nonsmooth systems consists in mechanical models with PWL nonlinearities, which may arise due to several effects such as damage or clearance [1]. In this work, the approach proposed in [2] for the use of singular perturbation theory on slow-fast PWL systems is extended from the autonomous configuration in $\mathbb{R}^3$ to nonautonomous two degrees of freedom slow-fast oscillators. Motivated by the quarter car model with bilinear damping characteristics [3], two examples of slow-fast PWL oscillators subjected to a harmonic excitation are used to illustrate the reduction in the case of bilinear damping and bilinear stiffness.



(a) Model. (b) FRC.

Figure 1: Model and frequency response curve of a 2-DOF slow-fast forced oscillator with PWL stiffness.

Results and discussion

In this contribution, a continuous matching of the locally invariant slow manifolds of a slow-fast forced system with piecewise linear nonlinearities is proposed. It was observed that the resulting reduced dynamics is able to approximate the behavior of the full system with high accuracy for a frequency range around the main harmonic. Due to the convergence property, the PWL system in the case of bilinear damping admits frequency-response curves (FRC) similar to a linear system, which were approximated with high fidelity by the proposed approach. For a similar PWL slow-fast oscillator with bilinear stiffness instead of damping, the system behavior becomes more complex due to the loss of the convergence property and the existence of nonlinear phenomena, such as super- and subharmonic resonances, becomes possible. These nonlinear resonances were accurately captured by the proposed reduction approach for the frequency range around the main resonance.

References

[1] Butcher, E.: Clearance effects on bilinear normal mode frequencies. Journal of Sound and Vibration 224(2), 305–328 (1999)
[2] Karoui, A.Y., Leine, R.I.: Analysis of a singularly perturbed continuous piecewise linear system. Proceedings of the 10th European Nonlinear Dynamics Conference - ENOC (2022)
[3] Silveira, M., Wahi, P., Fernandes, J.: Effects of asymmetrical damping on a 2 DOF quarter-car model under harmonic excitation. Communications in Nonlinear Science and Numerical Simulation 43, 14–24 (2017)