Fractional-Order Extension and FPGA Verification of Chaotic Models for Several Diseases

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Abstract. Among several definitions and numerical methods for fractional-order derivatives, Grünwald-Letnikov (GL) definition is suitable for numerical calculations and, hence, digital hardware realization. This paper extends several integer-order disease models to the fractional-order domain and verifies their chaotic behavior in software and hardware. The current paper considers Covid and Cancer disease models. All these models are nonlinear, with many terms and parameters. GL implementation adopts simplified steps to calculate the binomial coefficients, summation, and exponentiation of the numerical solution step size raised to the power of the fractional order. For some of the implemented systems, the nonlinear terms include division by the state variable, which is a challenging task in digital implementation. Consequently, the linear approximation method is applied, where the linear binomial coefficients are generated based on MATLAB curve fitting. Fractional-order cancer and Covid-19 models are realized experimentally on Xilinx FPGA Artix-7 XC7A100TCSG324, and the strange attractors are displayed on the Oscilloscope. The two systems operate at maximum frequencies of 33.009 and 2.259 MHz and yield throughputs of 0.396 and 0.267, respectively. The proposed models have more parameters than the corresponding integer models, which improves their modelling capabilities at the expense of slightly higher resource consumption and lower frequencies because of the relatively extensive arithmetic operations needed for the fractional system solution.

Introduction

Epidemics and infectious diseases modelling is a relatively difficult problem, since their dynamics vary largely from one outbreak to another. For such emerging and reemerging diseases, the causes and transfer processes are often poorly known and understood. Susceptible, infected, recovered and possibly also exposed classes of a given population need to be considered. Such models may exhibit chaotic dynamics for some parameter ranges. Fractional calculus enables differentiation and integration of arbitrary real order and describes the actual dynamics of natural phenomena more accurately than the special case of integer order [1]. Its applications have gained increasing attention in many fields including diseases and chaotic modelling. Grünwald-Letnikov definition is one of the most suitable fractional definitions for numerical calculations and hardware realization on Field Programmable Gate Arrays (FPGAs). FPGAs have many benefits, including ease of design, high speed, programmability, low hardware cost, noise immunity, rapid prototyping, reliability and reconfigurability. Fixed-point operations are widely used for hardware realizations to save costs and enhance speed.

Results and discussion

The proposed fractional Covid 19 and Cancer designs are coded in Verilog HDL, simulated on Xilinx ISE 14.7 and realized on Xilinx FPGA Artix-7 XC7A100TCSG324 using Chip scope. A summary of FPGA implementation results of this work and [2] is presented. The proposed models consume higher resources and lower frequencies than the integer peers in [2] because of the extensive arithmetic operations needed for fractional system solution compared to integer. On the other hand, the proposed models have more parameters than the corresponding integer models, which improves their performance in some applications such as encryption schemes [3]. The proposed realizations achieve throughputs of 0.267 and 0.396 Gbit/s for fractional Covid 19 and Cancer models, respectively.

References

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