Nonresonant interactions between a linear system and a light double limit cycle oscillator

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Abstract. We study modal interactions between a primary spring-mass-dashpot system and a light secondary system that can exhibit two limit cycles. The primary system with negative damping can represent a linearly unstable mode of a structure while the secondary system can act as an untuned stabilizer for this unstable mode. The coupled system is then studied using the method of multiple scales. Here, we obtain decoupled amplitude and phase evolution equations. The amplitude evolution equations are further simplified using a parameter-dependent transformation. We find that the system's qualitative dynamics depend only on two non-dimensional parameters. We then identify boundaries in this parameter space between regimes corresponding to different qualitative dynamics. Corresponding amplitude phase portraits allow us to identify system parameters that can offer large basins of attraction for the stabilized primary system.

Introduction

Engineering structures prone to instability, e.g., cables in cross-flow and long boring bars can be modeled as a primary spring-mass oscillator that is negatively damped. Traditional stabilization strategies include active damping which require massive actuators and/or passive tuned linear or nonlinear absorbers which require precise tuning [1]. In contrast, we consider an untuned stabilizer similar to [2] where they showed that an untuned whirling pendulum can stabilize an unstable mode. In the present work, we attach a light, nonresonant, secondary system that can exhibit two limit cycles: one stable and one unstable. We note that the attached system, being nonresonant, does not require precise tuning. Figure 1 (left) shows a schematic of the coupled system for which the final non-dimensional equations of motion are

$$\ddot{x} + \mu^2 c\dot{x} + x - \mu m\omega_p^2 z - \mu^2 m\gamma\dot{z}(\nu z^4 - z^2 + 1) = 0,$$
$$\ddot{x} + \ddot{z} + \omega_p^2 z + \mu\gamma\dot{z}(\nu z^4 - z^2 + 1) = 0,$$

where μ denotes a small bookkeeping parameter. Details of the nondimensionalization are omitted here. The two uncoupled limit cycles exist for $0 < \nu < 0.125$ with the larger amplitude cycle being stable for $\gamma > 0$.

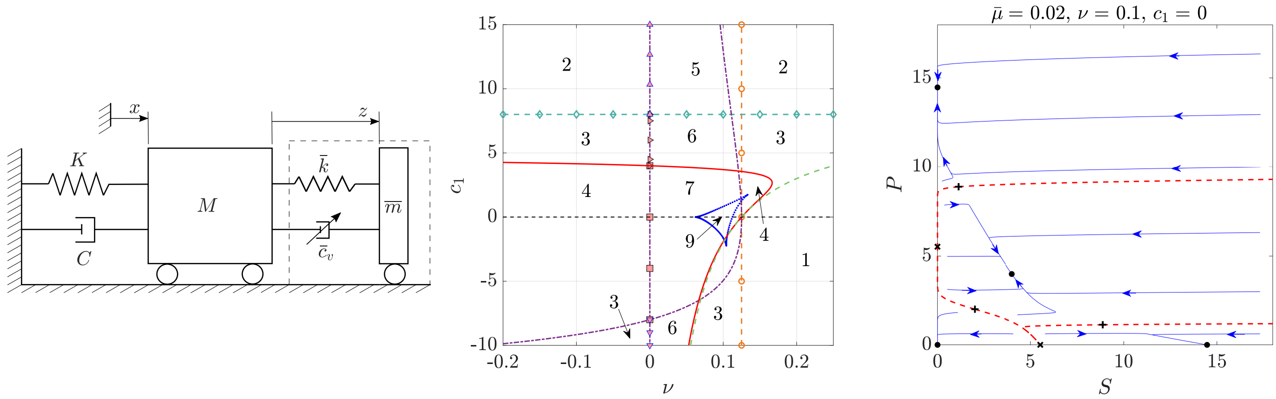

Figure 1: Left: system schematic with usual notations for parameters. Middle: the two parameter plane with number of fixed points in the amplitude plane. Right: the phase portrait corresponding to the c_1-ν regime with 9 fixed points. $\bar{\mu}$ scales time: S evolves faster.

Results and discussion

The method of multiple scales yields decoupled amplitude and phase evolution equations. That allows us to focus on the amplitude plane only. We transform the amplitude evolution equations into a simple form using a parameter-dependent transformation. The final equations are in terms of the variables P and S denoting squares of the amplitude of the primary and secondary oscillators, respectively. The equations reveal that the qualitative dynamics of the system is governed by two parameters: $c_1 = -\dfrac{8c(1-\omega_p^2)^2}{m\gamma}$ representing a measure of the ratio of the damping in the primary and the secondary system and ν, the ratio of the quartic and the quadratic terms in the nonlinear damping of the secondary system. We identify regimes with different number of P-S fixed points in the c_1-ν plane, see Fig. 1 (middle). The corresponding finite number of phase portraits present the unified system dynamics. Figure 1 (right) shows the one with the highest number of fixed points, clearly showing a stable state with $P = 0$ which represents a stabilized primary system. From this analysis, we can identify c_1-ν domains that provide large basins of attraction for the stabilized primary oscillator.

References

[1] Gattulli V., Di Fabio F., Luongo A. (2004) Nonlinear Tuned Mass Damper for self-excited oscillations. *Wind Struct.* **7**(4):251-264.

[2] Singla S., Chatterjee A. (2020) Nonlinear responses of an SDOF structure with a light, whirling, driven, untuned pendulum. *Int. J. Mech. Sci.* **168**:105305.