## Constrained Green's function for a beam with arbitrary spring and nonlinear spring foundation

X. Zhao\*, Q. Wang\*, W. D. Zhu\*\*, Y. H. Li\*\*\*

\*School of Civil Engineering and Architecture, Southwest Petroleum University, Chengdu, Sichuan, PR China

\*Department of Mechanical Engineering, University of Maryland, Baltimore County, Baltimore, USA

\*\*\* School of Mechanics and Engineering, Southwest Jiaotong University, Chengdu, Sichuan, PR China

Abstract. As an important structure of micro-robots, micro-beams play an increasingly important role in daily production and life, especially in the biological and medical fields. During the use of micro-beam instruments, vibrations occurs due to the unevenness of the skin, which will affect the accuracy and stability of precision instruments. To analyze this problem, this paper studies the free vibration of beams with spring at arbitrary position and nonlinear spring foundations. Through the Laplace transform and the principle of linear superposition, the constrained Green's function is obtained. Numerical calculations are performed to validate the present solutions and the effects of various important physical parameters are investigated. It was found the mode and deflection of the beam were changed by the spring.

## Introduction

The research on the dynamic behavior of beams has been a classic problem, which has attracted many scholars since its inception. As an effective tool, various forced vibration problems are studied through Green's function. In order to study the vibration characteristics of the beam, Li et al.[1] used Green's function to study the deflection and frequency of Timoshenk beam with damping. In order to analyze the interaction of double beam structures, Zhao et al.[2] used Green's function method to study the forced vibration of Timoshenko double beam system under axial compression load. Li et al.[3] proposed a Green's function solution of forced vibration of two parameters.

In a word, the previous Green's function method focused on the study of various forced vibration problems, but not on free vibration problems. In this paper, the free vibration of beams with arbitrary springs and nonlinear foundations is studied by applying the Green's function method. The effects of some physical parameters are studied and meaningful conclusions are obtained.

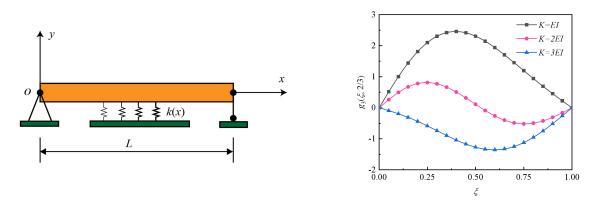


Figure 1: Nonlinear spring foundation.

Figure 2: Effect of spring stiffness on the constrained Green's function.

## **Results and discussion**

Figure 1 shows a simply supported beam with springs. There are a series of springs with varying stiffness at the bottom of the beam. In this paper, the free vibration problem is solved by Green's function method. It can be seen from Figure 2 that the mode is modified by the spring stiffness. As the spring stiffness increases, the mode of the constrained Green's function change accordingly.

## References

- Li X.Y., Zhao X., Li Y.H. (2014) Green's Functions of the Forced Vibration of Timoshenko Beams with Damping Effect. J. Sound Vib 333: 1781–1795.
- [2] Zhao X., Chen B., Li Y.H., Zhu W. D., Nkiegaing F. J., Shao Y. B. (2020) Forced Vibration Analysis of Timoshenko Double-Beam System under Compressive Axial Load by Means of Green's Functions. J. Sound Vib 464: 115001.
- [3] Li M., Zhao X., Li X., Chang X.P., Li Y.H. (2020) Stability Analysis of Oil-Conveying Pipes on Two-parameter Foundations with Generalized Boundary Condition by Means of Green's Functions. *Eng. Struct* 173: 300–312.