# The nature and formation of rogue waves for nonlinear Schrödinger equation

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## Abstract

We discuss the nature of rogue waves (RW), their ingrained instability and dynamic generation in systems governed by the standard cubic nonlinear Schrödinger equation (NLSE). We also discuss the spatiotemporal pattern of high intensity peaks in the form of multi-elliptic RW clusters.

## Introduction

Rogue waves are high-intensity nonlinear waves that suddenly appear and disappear without a trace in oceans and optics. They can be build up using Darboux transformation (DT) scheme from trivial, so called zero seed NLSE solution, using Lax pair formalism and recursive relations. The idea is to construct fundamental NLSE solution of various orders, known as Akhmediev breathers, analytically from DT. We can extract initial conditions from analytical solutions and generate high intensity peaks (rogue waves) dynamically.

## Results and discussion

We analyze the method of mode pruning for suppressing the modulation instability of RWs and to produce stable Talbot carpets by RWs (figure 1) that are recurrent images of light waves. We focus on cases when rogue waves appear as numerical artefacts, due to an inadequate numerical treatment of modulation instability. We further display how statistical analysis based on different numerical procedures can lead to misleading conclusions on the rogue waves nature [1].

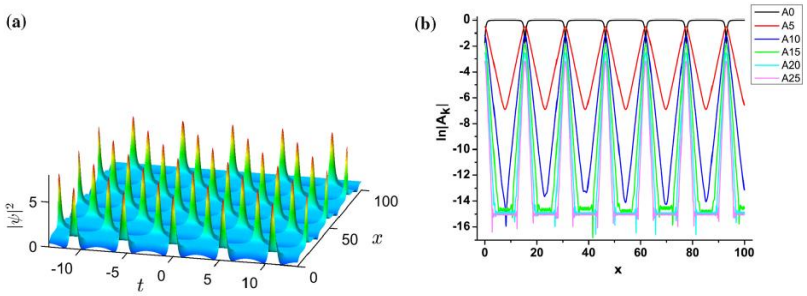


Figure 1: Double-periodic numerical solutions made of the first-order NLSE breathers, using the pruning procedure in the fast Fourier transform: (a) five breathers in the box with the pruning, (b) its spectrum.

The next research topic is the formation of rogue waves. We analyze the various spatiotemporal patterns of RWs which may have the form of multi-elliptic clusters (figure 2). Such structures may be obtained on uniform and elliptic dnoidal backgrounds using the DT scheme. We solve the eigenvalue problem of the Lax pair of order $n$ in which the first $m$ evolution shifts are equal, nonzero, and eigenvalue dependent, while all eigenvalues' imaginary parts are close to one. We show that an Akhmediev breather of order $n - 2m$ appears at the origin and can be considered as central rogue wave. We show that the high-intensity narrow peak, with the complex intensity distribution in its vicinity, is enclosed by $m$ ellipses consisting of the first-order Akhmediev breathers. The number of maxima on each ellipse is determined by its index and solution order [2].

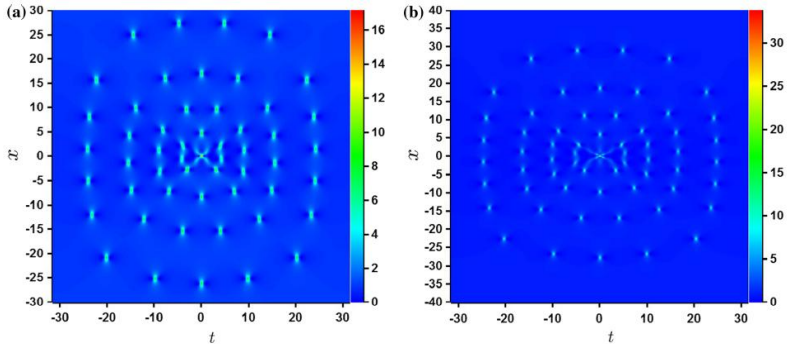


Figure 2: 2D color plots of rogue wave clusters on the uniform background having four ellipses ($m$ = 4) around $n - 2m$ order rogue wave, formed at the origin (0,0) of the $(x,t)$ plane. The orders of Darboux transformation and the Akhmediev breather representing the high-intensity central peak are: (a) $n$ = 10 with the second-order rogue wave, and (b) $n$ = 11 with the third-order rogue wave.

## References

[1] Belić M.R., Nikolić S.N., Ashour O.A., Aleksić N.B. (2022) On different aspects of the optical rogue waves nature. *Nonlinear Dynamics* **108**:1655-1670.

[2] Nikolić S.N., Alwashahi S., Ashour O.A., Chin S.A., Aleksić N.B., Belić M.R. (2022) Multi-elliptic rogue wave clusters of the nonlinear Schrödinger equation on different backgrounds. *Nonlinear Dynamics* **108**:479-490.