Effects of Prey and Predator Dispersal on a Discrete-time Population Model

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Abstract. We examine the effect of dispersal rates on a discrete-time homogeneous two-patch predator-prey model with prey and predator dispersal. An increase in either the prey or predator dispersal rate leads to the destabilization of the system by undergoing various bifurcations. The system shows complex dynamical behaviour such as quasiperiodicity, periodic windows, period-bubbling, and chaos as the dispersal rate is increased. Different stable attractors are present for different initial conditions indicating multistability in the system. We observe the increase or decrease in the mean population density of both species with respect to dispersal rate when the system is in non-equilibrium states.

Introduction

Dispersal plays an important role in ecology. Bajeux and Ghosh [1] investigated a metapopulation model to show that spatial heterogeneity induces stability switching and hydra effect. Vortkamp et al. [2] observed that the overall population is reduced with increase dispersal between two heterogeneous habitats of a single species. The increase coupling parameter in a discrete-time system of two coupled identical logistic maps leads to destabilization of the system (Bashkirtseva and Ryashko [3]). The effect of dispersal rates on any discrete-time predator-prey model is of great interest in estimating the population density of the species and stability of the system. Hence, we analyze the following system which could be derived using Euler's method from a continuous-time model similar to Bajeux and Ghosh [1]:

$$x_{n+1} = x_n + r x_n\left(1-\frac{x_n}{K}\right) - \frac{\alpha x_n u_n}{h+x_n} + d(y_n - x_n), \quad u_{n+1} = u_n + s u_n\left(1-\frac{u_n}{L}\right) + \frac{\beta x_n u_n}{h+x_n} + D(v_n - u_n),$$

$$y_{n+1} = y_n + r y_n\left(1-\frac{y_n}{K}\right) - \frac{\alpha y_n v_n}{h+y_n} + d(x_n - y_n), \quad v_{n+1} = v_n + s v_n\left(1-\frac{v_n}{L}\right) + \frac{\beta y_n v_n}{h+y_n} + D(u_n - v_n),$$

with initial population $x_0 > 0$, $u_0 > 0$, $y_0 > 0$, and $v_0 > 0$. Here, x_i (respectively, y_i) is prey population and u_i (respectively, v_i) is predator population in the ith generation in patch 1 (respectively, 2). The parameter r and s are intrinsic growth rate of prey and predator species, respectively. The carrying capacities of prey and predator species are denoted by K and L, respectively. The predation coefficient is represented by α. Here, $\beta = \alpha c$, where c is the conversion coefficient of prey biomass to predator biomass. The parameter h is the half-saturation constant. The dispersal rates of prey and predator species are d and D, respectively.

Results and Discussion

The equilibrium points of the sytem do not depend on the prey or predator dispersal rates. The stability of the equilibrium points is determined using Routh-Hurwitz conditions. **Fig. 1(a)** shows that first, the system is stable, then undergoes period-doubling bifurcation leading to a stable two-period cycle. Neimark-Sacker bifurcation is encountered by the stable two-period cycle where two-stable closed orbits arise, leading to chaos through quasiperiodicity. The system eventually settles into chaotic behavior after exhibiting complex phenomena like periodic windows and period-bubbling as the prey dispersal rate increases further. Multistability is evident from **Fig. 1(b)** where we plotted bifurcation diagrams using two different initial conditions. The system is multistable wherever the red and blue colored bifurcation diagrams don't coincide. The mean prey (predator) density is constant with increase in prey dispersal rate when the system is stable. However, in the non equilibrium states of the system, the mean densities of both the species are decreasing for intermediate prey dispersal rate, and the densities are increasing when the dispersal is increased sufficiently (**Fig. 1(c)-(d)**). Similar results are obtained by increasing the predator dispersal rate.

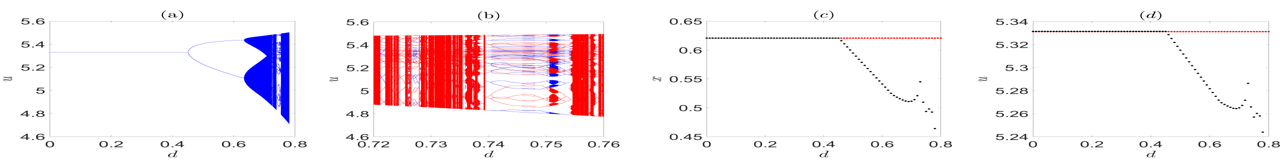

Figure 1: Taking $r = 2$, $s = 1$, $K = 1$, $L = 5$, $\alpha = 0.8$, $\beta = 0.6$, $h = 5$, and $D = 0.1$ (a) bifurcation diagram of the predator population in patch 1 with d, (b) bifurcation diagram with two initial conditions: $(0.5, 5, 0.52, 5)$ in blue colour and $(0.5, 4.85, 0.52, 5)$ in red colour, (c) mean prey density in patch 1 with respect to d, and (d) mean predator density in patch 1 with respect to d.

References

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