Oscillations of a Nonlinear Beam in Contact with a Rigid Cylindrical Constraint

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Abstract. In this work we consider the free oscillations of a fixed-free nonlinear Euler Bernoulli beam in contact with a rigid cylindrical constraint. In addition to the geometric nonlinearity due to deformation, the unilateral constraint introduces nonlinearity due to change in effective beam length as it deforms. We explore the existence of nonlinear normal modes (NNMs) and their dynamics in this dynamical system invoking the Galerkin method, method of averaging and FE methods.

Introduction

Energy dependent frequency of oscillations of a pendulum is well known. Owing to Huygens (1656) ingenuity of varying the effective length of string as it wraps/unwraps around (cycloidal) foundation, the oscillations were rendered isochronous. In the late twentieth century, researchers started investigating the dynamics of flexures as they wrap/unwrap around obstacles. Fung et al. [1] considered Euler-Bernoulli beam with rigid cylindrical foundation on one side and investigated the effect of the resulting nonlinear transversality condition. Crespo da Silva et al. [2] developed a nonlinear Euler-Bernoulli beam model with inextensibility constraint and explored the free and forced nonlinear oscillations and the resonances thereof. In the current study we consider the nonlinear beam [2,3] coming into contact with a unilateral cylindrical constraint (Fig. 1).

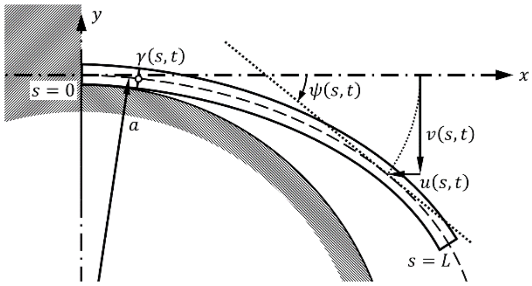

Figure 1: Kinematics of the nonlinear beam.

Discussions

To study the conservative dynamics of the beam, we use the principle of noncontemporaneous variation resulting in the governing equation Eq. (1), where notations and symbols have usual meaning. In this study we consider point-wise inextensibility of the beam and introduce Lagrange multiplier to incorporate the corresponding constraint. Considering a fixed-free boundary condition, the beam wraps around the cylinder such that u and v conform to the constraint $(s+u)^2+(a+v)^2=a^2$ over the region $0 \le s \le \gamma(t)$. The axial force in the beam over this domain is constant and is equal to $F=-EIv_{sss}(\gamma(t))v_s(\gamma(t))$. However, the axial force (Eq. (2)) in the subsequent domain $\gamma^+(t) < s \le L$ is time varying such that the force at the free end is zero. In essence, such a varying axial force is required to maintain the overall inextensibility of the beam.

$$\rho A v_{tt} + EIv_{ssss} + EI\{v_s(v_s v_{ss})\}_s + \frac{\rho A}{2}\left\{v_s \int_L^s \left(\int_0^s v_s^2\, ds\right)_{tt} ds\right\}_s = 0 \tag{1}$$

$$F = -EIv_{sss}v_s - \frac{\rho A}{2}\int_L^s \left(\int_0^s v_s^2\, ds\right)_{tt} ds \tag{2}$$

The transversality condition to be satisfied at the last point of contact ($\gamma(t)$) corresponds to the difference between a combination of v_{sss} and its lower derivatives before and after this point of contact. Owing to the complexity of the problem, our analytical study considers a combination of Galerkin method, method of averaging and finite element methods to explore the existence of nonlinear normal modes [4,5] and their stability thereof.

References

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