Stochastic delay modelling of landslide dynamics

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Abstract. In present paper we provide results on qualitative modelling of landslide dynamics under the assumption of delayed failure and the effect of background noise. Results obtained indicate that examined time series, which represent the actual recordings of background noise, belong to a group of stationary linear stochastic processes with Gaussian inputs. Such noise is introduced as the additive term in the system of delay differential equations governing the dynamics of spring-block model composed of n units with delayed failure. Friction law assumed in the model represents the cubic friction force. Results of the performed research, using mean-field approximation and numerical computation, indicates the conditions for occurrence of Andronov-Hopf direct supercritical bifurcation. In particular, it is shown small-amplitude background noise could contribute to the onset of in-stability if the system under study is at the verge of stability.

Introduction

In present paper our analysis is based on the model initially proposed by Davis [1], who assumed that dynamics of accumulation slope could be qualitatively described by the interaction of feeder and accumulation part of the slope. The existence of noise is firstly evaluated using surrogate data testing of the two null hypotheses: that data are independent random numbers, and from a stationary linear stochastic processes with Gaussian inputs. Noise is proposed to be introduced as additive white noise as a part of the system of delay differential equations governing the dynamics of spring-block model composed of n units with delayed failure, which is assumed to qualitatively mimic the landslide motion [1]:

$$x_i(t) = y_i(t)dt$$
$$dy_i(t) = -\left[a\left(V+y_i(t)\right)^3 - b\left(V+y_i(t)\right)^2 + c\left(V+y_i(t)\right)\right]dt + (aV^3 - bV^2 + cV)dt +$$
$$+\sum_{j=1}^{N} k\left(x_j(t-\tau) - x_i(t)\right)dt + \sqrt{2D}dW_i \qquad (1)$$

where x_i and y_i are displacement and velocity of the i-th block, V=0.2 is the constant background velocity of the system, τ is the introduced time delay and k is the spring stiffness. Terms$\sqrt{2D}dW_i$ represent stochastic increments of independent Wiener process, i.e. dW_i satisfy: $E(dWi) = 0$, $E(dW_i dW_j) = \delta_{ij}dt$, where *E()* denotes the expectation over many realizations of the stochastic process and D is intensity of additive local noise. Friction law assumed in (1) represents the cubic friction force, with frictional parameters *a*, *b* and *c* as proposed in [2]. As it was shown in previous research [3], noise could represent the main trigger for instability to occur for certain initial conditions or dynamical regime of the system under study.

Results and discussion

In the first phase of the research, surrogate data testing is invoked to examine the recorded background seismic noise [4]. Results obtained indicate that zeroth-order prediction error for the examined time series is smaller than zeroth order prediction error for 20 surrogate time series, which were obtained by random shuffling of the starting data. On the other hand, it seems that the observed time series could be characterized as stochastic linear series with Gaussian inputs, since the results show that distribution of zeroth order prediction error for the starting series with the increase of prediction steps is well within the prediction error for the surrogate tests. In the second part of the research, we examined the dynamics of the proposed model (1). System is analyzed for the following parameter values: a=3.2, b=7.2 and c=4.8, V=0.2, and for the following initial conditions: $x_{10}=0.001$, $x_{20}=0.0001$, $y_{10}=0.1$, $y_{20}=0.1$. Results of the performed research, using mean-field approximation and numerical computation, indicates the conditions for occurrence of Andronov-Hopf direct supercritical bifurcation. In particular, it is shown small-amplitude background noise could contribute to the onset of instability if the system under study is at the verge of stability. Moreover, results obtained indicate conditions for which the decrease of friction force and delayed failure leads to onset of instability, as well the conditions for which the increase of friction force leads to stabilization of sliding.

References

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