Physics Enhanced Sparse Identification of Nonlinear Oscillator with Coulomb Friction

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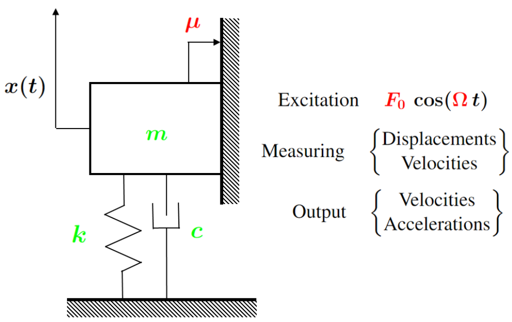
Abstract. This study investigates the identification of the nonlinear governing equations of a Single-Degree-of-Freedom oscillator under harmonic excitation, including Coulomb friction damping. The approach used for this task is the so-called RK4-SINDy [1], enhanced by incorporating part of the known physics. This simple, yet representative case study, is examined using both artificially generated noisy data, and data obtained from an experimental setup. The obtained results, highlight the potential of this framework in nonlinear system identification, given sparsely collected corrupted data, and the benefits of incorporating already known system information.

Introduction

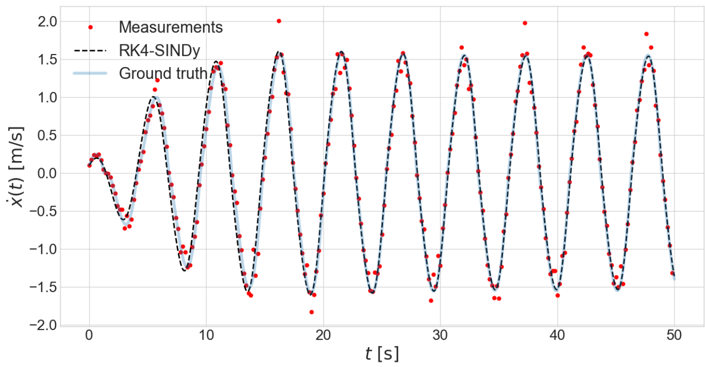
Frictional joints are present in a plethora of applications and fields, such as the aerospace, automotive, and building industries. Therefore, an ever-important challenge in the analysis of engineering systems is the understanding of friction damping in structural dynamics. Due to its nonlinear and non-smooth nature, the available approaches, including proposing alternative constitutive laws and validating models based on experimental data [2], cannot deal with the identification of friction forces. One way around this is to take advantage of the rapid increase of data availability through measurements for complex engineering systems, and newly developed identification techniques of the underlying differential equations of physical problems based on noisy measurements. A promising framework called Sparse Identification of Nonlinear Dynamics (SINDy) [3] was developed for this purpose, aiming to derive parsimonious solutions of nonlinear systems, which was further improved by combining the principles of dictionary-based learning, which is a key concept in SINDy, with numerical analysis tools, and more specifically the 4th order Runge-Kutta integration scheme [1]. This approach, the so-called RK4-SINDy was proven to be more efficient when dealing with noisy and sparsely collected data, not exploring though the incorporation of physics in discovering nonlinear models.

Results and discussion

The identification of the governing equations of an SDOF oscillator, including harmonic excitation and Coulomb friction damping is investigated (Figure 1a). Part of the system's equation of motion is assumed known during the identification of the vector field of the global response, incorporating in this way part of the known physics, while regarding the input data, noisy measurements (both artificially generated and experimental) were used.



(a) Problem schematic representation (unknown quantities in red, known quantities in green).



(b) Velocity identification from artificially generated noisy measurements.

Figure 1: Problem description - Results

It is evident (Figure 1b) that this approach leads to accurate results even for significant noise levels while maintaining a parsimonious solution in order to avoid overfitting the noisy data.

References

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