Computation of the Wright function from its integral representation

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Abstract. The contribution presents a novel computational technique for the Wright function using numerical quadratures.

Introduction

The Wright function, introduced by E.M. Wright, is a special mathematical function originally defined by the infinite series:

$$W\left(a, b \middle|\, z\right) := \sum_{k=0}^{\infty} \frac{z^k}{k!\,\Gamma(ak+b)}, \quad z, b \in \mathbb{C}, \quad a > -1$$

The Wright function provides a unified treatment of several classes of special functions, notably the Bessel functions, the error function erf, the Airy function Ai, and the Whittaker function [1]. It is also related to the derivatives of the Gaussian $\exp\left(-x^2/2\right)$ and the Airy functions. The Wright function arises in the theory of the space-time fractional diffusion equation with the temporal Caputo derivative. Therefore, methods of its computation can be of general interest. Previous studies [2] treated only the case $|b| \leq 1$.

Results and Discussion

The present contribution removes this restriction and strives for full generality of the computational technique, which covers many applications, such as, for example, the computation of the Prabhakar and Mittag-Leffler functions. This is achieved by the use of the complex integral representation of the function along the Hankel contour encircling the negative real semi-axis:

$$W\left(a, b \middle|\, z\right) = \frac{1}{2\pi i} \int_{Ha^-} \frac{e^{\xi + z\xi^{-a}}}{\xi^b} d\xi$$

using the method of stationary phase. The algorithm is implemented as a standalone library using the double-exponential (DE) quadrature integration technique [3] in the Java programming language and can be downloaded from `https://github.com/dprodanov/dspquad`. A reference Maxima implementation was developed both for QUADPACK [4] and for DE libraries. Function plots are presented in Fig. 1 (left – negative $|a| < 1$, right – $a = 1$).

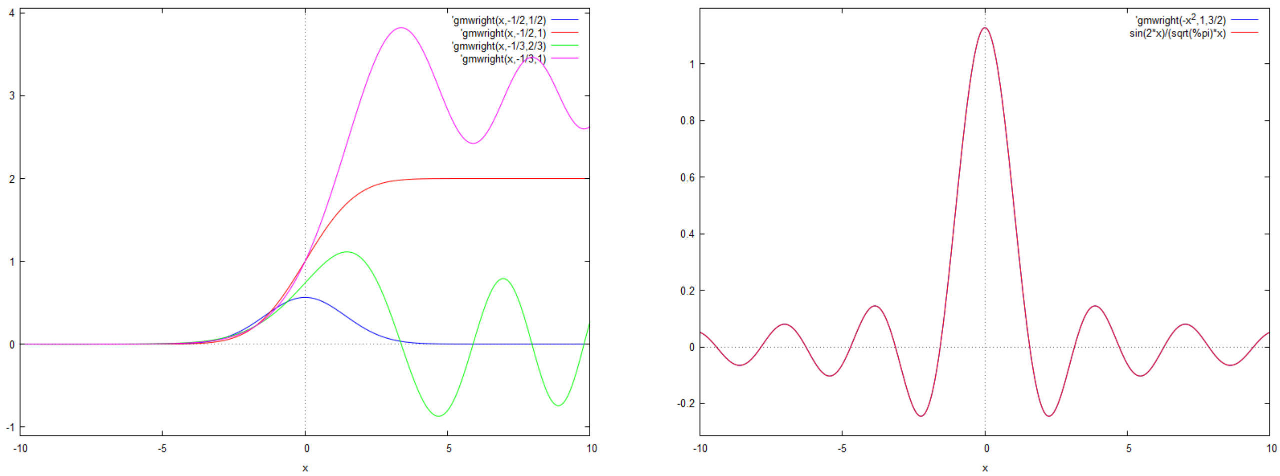

Figure 1: Wright function plots

References

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