Bandgap formation study of a geometrically nonlinear metamaterial

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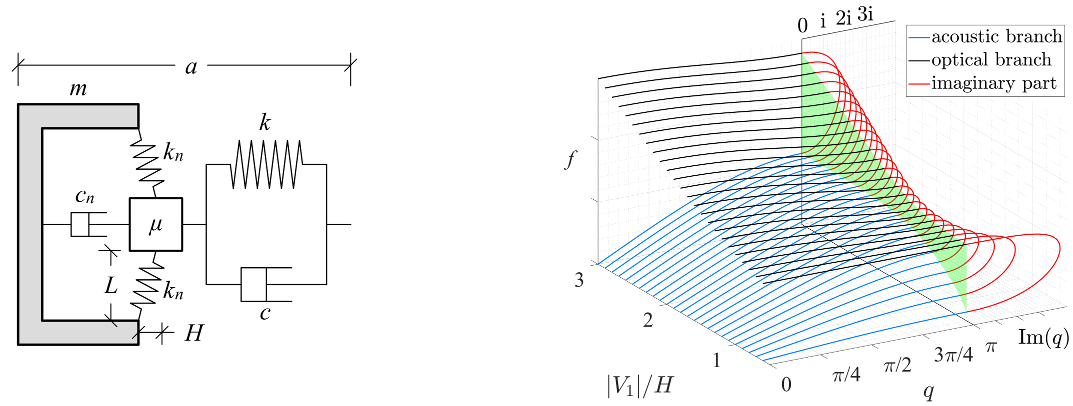
Abstract. The concept of metamaterials can be enhanced with use of nonlinear designs for vibration mitigation purposes. To specify the frequency zones within which wave propagation is prevented, it is necessary to study the corresponding dispersion curves. In this work, a geometrically nonlinear metamaterial inducing a negative stiffness effect is investigated. The bandgap formation is studied with the use of analytical methods and numerical techniques, evaluating their compliance. An energy dependent behaviour is identified, which is attributed to the nonlinear nature of the design.

Introduction

The mitigation and control of mechanical vibrations forms a major challenge in engineering applications. The recently developed concept of metamaterials can be applied for wave manipulation purposes [1]. These are configurations that consist of periodically arranged unit cells and can form specific frequency ranges (bandgaps), where wave propagation is obstructed. The introduction of nonlinearity in such configurations can enhance their vibration mitigation capabilities both in terms of widening the bandgap, as well as shifting this toward lower frequencies. The current study focuses on a geometrically nonlinear metamaterial lattice, inducing negative stiffness behaviour [2]. This is formed by periodic arrangement of repeated unit cells, as depicted in Fig. 1(a). Each cell, of lattice constant a, consists of an external mass m that is connected to an internal mass μ via a nonlinear element and linear viscous damping c_n, while neighboring cells are interconnected with linear stiffness and damping k and c respectively. The adoption of a triangular arch configuration results into geometric nonlinearity under large displacement consideration, while negative stiffness is observed in a specific region of the equilibrium path. This arch is formed by two identical linear springs of stiffness k_n, arranged in a triangular geometry of height H and base $2L$, as shown in Fig. 1(a).



(a) (b)

Figure 1: Geometrically nonlinear metamaterial. (a) Unit cell. (b) Dispersion curves (bandgap highlighted in green).

Results and discussion

Determination of the created bandgap requires calculation of the corresponding dispersion curves of an infinite lattice, i.e. the plot of the frequency f versus wave number q, depending on of the response magnitude. This calculation is initially performed via use of the Harmonic Balance Method (HBM), with the results of this process shown in Fig. 1(b). Due to the nonlinear nature of the design, the dispersion relation is dependent on the relative oscillation amplitude $|V_1|$, between the nonlinearly connected masses. The resulting bandgap is defined to be the range, where no real frequency solutions exist. To further study the bandgap characteristics of the system, the Nonlinear Normal Modes of the unit cell are evaluated for varying energy levels. Energy dependent curves are therefore formed for the in-phase and out-of-phase modes, indicating the boundaries of the bandgap [3]. The respective results are compared against the output of the HBM, evaluating the resulting agreement, while estimation of the dispersion curves with numerical techniques validates the analytical calculations.

References

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