On the influence of external damping on the dynamics of a generalized Beck's beam

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Abstract. The influence of external damping on the dynamic behaviour of a generalized Beck's beam is discussed in this work. The cantilever beam is inextensible and shear-undeformable, it is subject to the action of distributed, linear and nonlinear external damping (modelling the interaction with the surrounding environment), and to follower and dead loads applied at the free tip. Hopf bifurcation conditions are detected after a stability analysis of the initial straight configuration. Subsequently, the integro-differential equations of motion of the system are studied via a multiple-scale approach to assess the influence of the external damping on the limit-cycle that may occur once the Hopf critical load is overcome.

Introduction

According to the Ziegler Paradox, the introduction of a positive, vanishing linear damping into a circulatory system can lower the Hopf bifurcation load [1]. The visco-elastic Beck's beam [2], namely a cantilever beam with distributed damping and a follower load at the free tip, is a paradigmatic example of a continuous system that is subject to such a paradox. Its linear behaviour has long been studied to increase the Hopf critical load and to improve the beam stability via external damping devices [3]. The nonlinear behaviour of beams of this kind with lumped devices at the free tip is addressed in [4] to investigate the system behaviour around the double-zero bifurcation. The influence of a distributed nonlinear hysteretic damping (of the internal type, due to material properties) on the post-critical scenario around the Hopf bifurcation is discussed in [5]. Here, we address the nonlinear dynamic behaviour of a generalized Beck's beam, which is subject to the action of distributed nonlinear external damping, plus follower and dead loads at the free end. The influence of the parameters associated with different nonlinear damping types are discussed. Assuming small nonlinearities, the bifurcation equation that governs the system dynamics around the Hopf bifurcation is studied. The semi-analytical results obtained in this way are compared with those derivable via numerical approaches. These latter are also employed to address the case of moderately large nonlinearities.

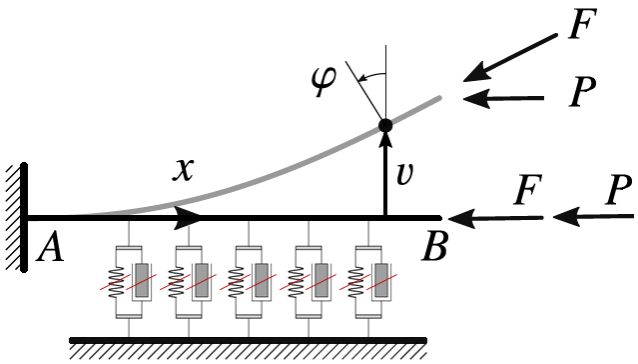

Figure 1: Schematic of the problem.

Results and discussion

The present study has shown that the constitutive characterization of the external damping can have a strong influence on the critical and post-critical behaviour of a generalized Beck's beam. In particular, the choice of the parameters associated with the nonlinear part of the damping is crucial and may induce beneficial or detrimental effects on the system nonlinear response. Comparisons with the results of purely numerical approaches confirm the theoretical findings obtained via the multiple-scale algorithm developed within this study, which turns out to be a reliable tool for nonlinear dynamic analyses. This work represents a first step towards a more reliable design of devices to control the detrimental effects of the nonlinear damping.

References

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