# On the asymptotical description of solutions to the matrix modified Korteweg-de Vries equation

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**Abstract**. Noncommutative integrable systems extend classical soliton equations like the Korteweg-de Vries or the Nonlinear Schrdinger equation to the level of equations in (at least) matrix-valued functions. Characteristic properties - like nonlinear superposition - reemerge, but usually in more complicated form, which requires deeper investigation. The results represent a further development of the study in [3] and [4] which answers to the question to understand the asymptotical behaviour of special cases of matrix solutions of the mKdV equation.

## Introduction

The equations under investigation is the matrix mKdV equation: $V_t = V_{xxx} + 3\{V^2, V_x\}$, where capital case is used to denote that the unknown are, in general, operators and, hence, non commutativity property is assumed. In the present investigation the attention is focussed on the case when $V$, is represented by a $2 \times 2$ matrix. Motivated by explicit solutions given in [4] with striking interaction behaviour, the aim of the present work is to contribute to a better understanding of 2-soliton solutions to the matrix modified Korteweg-de Vries (mKdV) equation by providing an explicit asymptotic analysis for solutions with regular spectral matrices (in the sense of [10]). The main tool is a parameter-dependent formula for the N-soliton solutions to the matrix modified Korteweg-de Vries equation given in [7] (see also [3] for an explicit proof). An asymptotic description of the 2-soliton solutions (for the $\mathsf{d} \times \mathsf{d}$-matrix mKdV) in the case that the spectral matrices are invertible. An example is provided in the two pictures where the soliton behaviour emerges.

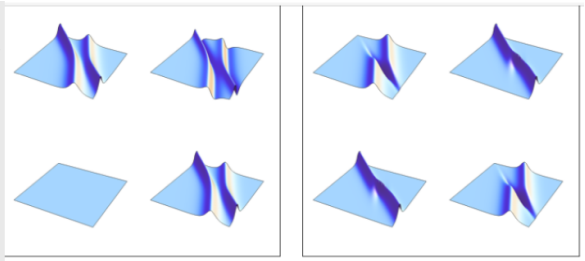

FIGURE 1. The solutions in Example 1 are depicted for $-10 \le x \le 10$ and $-5 \le t \le 5$ with plot range between $-\sqrt{2}$ and $\sqrt{2}$, the solution in **a)** to the left, the solution in **b)** to the right.

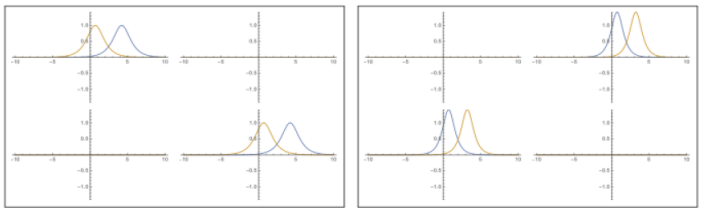

FIGURE 2. The solution from Example 1 **a)** is plotted for $t = -20$ (the brown curve) and $t = 20$ (the blue curve). The plot to the left is ploted in the frame $(x - t, t)$, the plot to the right i $(x - 2t, t)$, both for $-10 \le x \le 10$ with plot range between $-\sqrt{2}$ and $\sqrt{2}$.

As a result, we obtain - precisely as in the scalar case - that the collision is elastic and only results in a phase-shift. This is in contrast to interactions involving a non-invertible spectral matrix, which may in addition lead to a change of the asymptotic appearance of the interacting waves.

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