Vibration control of lossless transmission lines with nonlinear terminators: a simplifying approach

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Abstract. This article proposes mathematical modeling and a simplifying control approach for the nonlinear vibrations of lossless transmission lines. One extremity of the transmission line is under a nonlinear external force while the control force is applied on the other extremity. The model is formulated in terms of Neutral-type Delay Differential Equations (NDDEs). These equations are derived directly by applying D'Alembert's method to the Partial Differential Equation (PDE) governing the lossless transmission line. An approach is developed to simplify the control problem to a simple Retarded Delay Differential Equation (RDDE) with input delay rather than an NDDE. It is shown that a sufficient condition for the NDDE to be stabilized is the existence of a control input which stabilizes the simplified RDDE in the sense of input-to-state stability.

Introduction

In general, a transmission line is a specialized cable or other structure designed to conduct a signal. In electrical engineering, the conducted signal is electromagnetic waves, while in mechanical engineering, it may be mechanical waves such as elastic waves. The governing equation representing a lossless transmission line is given by the following PDE:

$$\frac{\partial^2 f}{\partial t^2}(x,t) = c^2 \frac{\partial^2 f}{\partial x^2}(x,t), \tag{1}$$

where $f(x,t)$ is a functon of the variables x and t and c is a constant. The wave equation (1) can be solved by using d'Alembert methods through introducing Riemann variables [1], which ultimately transforms the PDE in (1) to an NDDE. Such mathematical modelling appears in many practical engineering applications such as coaxial cable, chemical engineering reactor applications, ship stabilization, large-scale integration systems, and drilling systems [2]. Generally, three kinds of delays are involved in such NDDE systems, i.e., (i) the state delays, (ii) the delays in the argument of state derivatives (neutral-type delays), and (iii) the input delays. In recent years, there has been a great deal of focus on the control issue of nonlinear systems with input delay [3]. On the other hand, there is no similar study on the control issue of nonlinear neutral-type time delay systems with input delays. In this paper, the control problem of the nonlinear vibrations of a lossless transmission line is addressed. The equations of motion are formulated in terms of NDDEs with constant neutral-type and input delays. For a general case (the general nonlinear load), it is shown that designing a controller for such neutral-type delay dynamics is equivalent to stabilizing a more straightforward dynamics without neutral-type delay terms. In the presense of neutral-type delay terms, there exist vertical eigenvalue asymptotes in the system spectrum, which cannot be changed by any feedback control law. This is the main reason that makes the control design challenging for NDDEs. The proposed approach makes the control design simpler by providing a condition to design a controller for an alternative dynamics (rather than the NDDEs) without neutral-type delay terms, and consequently, in the absence of the vertical eigenvalue asymptotes.

Results and discussion

A novel control approach is presented to simplify the control problem of nonlinear vibrations of a lossless transmission line, formulated in terms of NDDEs. It is shown that by introducing an input transformation, the control problem of the NDDEs is simplified to an alternative control problem for a system without neutral-type delay terms. Indeed, it is proved that showing the input-to-state stability of the simplified system guarantees the asymptotic stability of the original NTD. The proposed approach is applied to a drill-string as a practical example of transmission lines and the stability is obtained by designing a controller for the simplified system.

References

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