Model Order Reduction of Nonlinear Thermal Systems using DEIM

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Abstract. A Proper orthogonal decomposition and hyper-reduction-based model order reduction method (MOR) is employed to obtain the computationally efficient solution of nonlinear thermal systems in one- and two-dimensional domains. Material nonlinearity, as well as nonlinearity due to boundary conditions, are considered. The hyper-reduction approach based on POD and Discrete Empirical Interpolation Method (DEIM) is used to solve the nonlinear system and compare the computational performance with the full finite element model.

Introduction

The finite element discretization of nonlinear partial differential equations leads to large scale nonlinear ordinary differential equations. The solution of these equations in time is computationally challenging. To obtain the computationally efficient solutions, model order reduction techniques are employed. The projection-based Proper orthogonal decomposition methods [1] are popularly used for model order reduction of nonlinear systems. However, to obtain the nonlinear matrices in lower dimensional subspace one has to first compute these matrices in the original dimensional space and then do the reduction which adds extra computations. To tackle this issue hyper-reduction methods such as Discrete Empirical Interpolation Method (DEIM), Energy-conserving Sampling and Weighing method (ECSW), etc. are proposed [2].

In this work we employ the POD-DEIM based MOR method to solve nonlinear heat transfer problems in one dimension and two dimensions. We consider the material nonlinearity and nonlinearity due to boundary conditions. The governing equations of the 2D system in the consideration is given in Eq. (1),

$$\frac{\partial}{\partial x}\left(\kappa(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa(T)\frac{\partial T}{\partial y}\right) = \rho C_p(T)\frac{\partial T}{\partial t} \tag{1}$$

The resulting system of nonlinear first order ODEs after FE discretization is given in Eq. (2),

$$\mathbf{C}(\mathbf{T})\dot{\mathbf{T}} + \mathbf{K}(\mathbf{T})\mathbf{T} = f \tag{2}$$

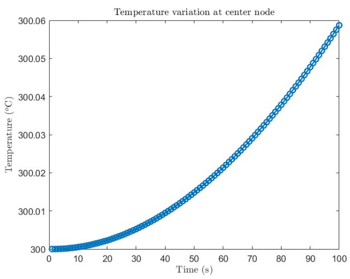
where, $\mathbf{K}(\mathbf{T})$ and $\mathbf{C}(\mathbf{T})$ are the nonlinear conductivity and capacitance matrices, respectively. Let $\mathbf{\Phi}$ be matrix consisting of POD modes for the problem under consideration. The reduced order model obtained by the POD method is given by,

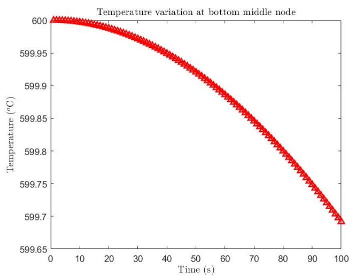
$$\mathbf{C}_r(\mathbf{T})\dot{\mathbf{T}} + \mathbf{K}_r(\mathbf{T})\mathbf{T} = f_r \tag{3}$$

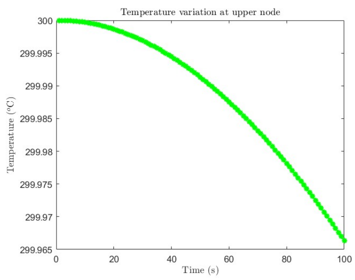
where, $\mathbf{C}_r = \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}$, $\mathbf{K}_r = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}$ and $f_r = \mathbf{\Phi}^T f$ are the reduced matrices. The computational complexity of Eq. (3) is further reduced by implementing DEIM algorithm.

Results and Discussions

We carry out the numerical experiment of 1D and 2D heat transfer problem with temperature dependent thermal conductivity and heat capacity. We consider Dirichlet and convection boundary conditions. In both one- and two-dimensional systems, DEIM gives more than 20 times faster computation compared to the full order model and 8 times speedup with respect to POD-based model, while maintaining accuracy of around 99%. Following are temperature variations at a few selected nodes:







References

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[2] Chaturantabut, S. and Sorensen, D. C. Nonlinear model reduction via discrete empirical interpolation. SIAM Journal on Scientific Computing. 32 (5), 2737-2764 (2010).