Koopman operator based Nonlinear Normal Modes for systems with internal resonance

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Abstract. This study is focused on the Nonlinear Normal Modes (*NNMs*) of a 2-dof nonlinear system with an essentially nonlinear substructure. The primary system comprises of an oscillator with cubic stiffness and is coupled with an essentially nonlinear oscillator with cubic stiffness. The energy flow from primary oscillator to the attachment is investigated by quantifying the *NNMs* of the system, by adapting the Shaw-Piere *invariant manifold technique*. Later the issues regarding the invariant manifold formulation is discussed and solved by introducing *Koopman Operator*.

Introduction

Nonlinear Normal Modes *(NNMs)* for nonlinear dynamical systems, developed as a generalization of Linear Normal Modes *(LNMs)*, are invariant manifolds in the state space that are tangent to the modal plane of LNM at the equilibrium points. Unlike LNMs, NNMs need not be linearly independent and therefore do not form the basis and can exceed the degrees of freedom of the system in number. Shaw *et. al.* [1] defined NNMs as *invariant manifolds* of dimension 2, embedded in the phase space and extended the idea to non-conservative systems [2]. While there has been significant progress on development of NNMs, the formulation of invariant manifolds for systems with *internal resonating modes*, especially systems with essential nonlinearity appears to be challenging.

This study focuses on formulating the invariant manifold (NNM) by adapting the technique proposed in [1] for a targeted energy transfer problem comprising of a 2-dof nonlinear system with an essentially nonlinear substructure. The primary system is taken to be an oscillator with a cubic stiffness. Identifying the NNMs enable quantifying the unidirectional flow of energy from the primary structure to its attachment. Three different cases, comprising of 3^{rd} order, 5^{th} order and 7^{th} order approximation of the invariant manifold is considered. It is shown that the invariance property is not global on the manifold due to internal resonance. The formulation of invariant manifold is then modified by introducing *"Koopman Operator"* [3], which enables dealing with effects of the complex geometry of folding of the invariant manifolds due to internal resonance.

Results and discussions

Three different order approximations of invariant manifold of a system comprising of a Duffing oscillator attached with a *Nonlinear Energy Sink (NES)*, is shown in Fig. 1. Here, (x_1, y_1) represents the state variables ("master co-ordinates") of the primary Duffing oscillator, while X_2 represents the displacement of the NES. It is observed that the invariance property holds locally. The deviation of the higher energy trajectories from the invariant manifold is observed to decrease significantly by increasing the order of the approximation.

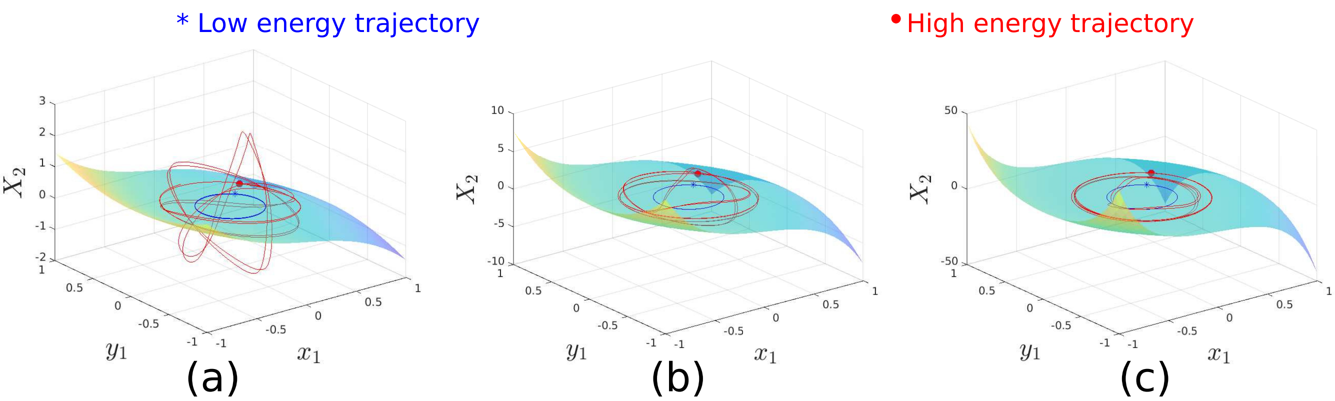

Figure 1: Invariant manifold of Duffing-NES system: (a) 3^{rd} order, (b) 5^{th} order and (c) 7^{th} order approximations.

References

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